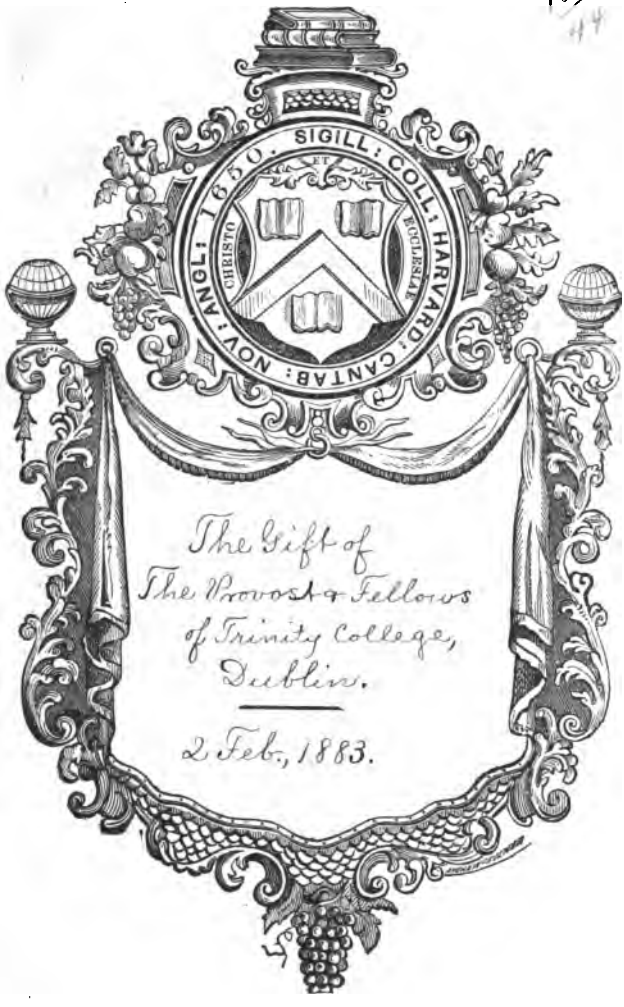
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AN INTRODUCTION TO LOGIC.

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AN INTRODUCTION TO LOGIC.

BY

William Henry Stanley

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PREFACE.

THE first ten chapters of the following treatise are intended for beginners, and nine of them are reprinted with but slight alterations from a series of articles on Logic contributed by me to *Our School Times*, a periodical then attached to Monaghan Diocesan School, but now to Foyle College, Londonderry. They were written at the request of Dr. Hime, the Head Master, who was also the Editor of the periodical in question, with the object of making the subject intelligible to advanced schoolboys and other beginners who might be among the readers of the paper. Those who desire to become acquainted with the fundamental principles of Deductive Logic only may confine their attention to these chapters; but it will be seen that the supplemental chapters intended for those who desire to carry their studies farther (which now appear in print for the first time) occupy about two-thirds of the book. I do not think the order in which the various matters treated of are dealt with will occa-

sion any embarrassment to the student who reads on to the end; while for him who means to stop at the elementary principles no other order would have been suitable.

I do not lay claim to originality in any part of the following treatise, nor indeed is originality possible in the case of a writer who takes the view of Logic which I have done—except perhaps in the criticism of hostile systems. The treatise which I now present to the public is a compilation, but it claims to be the work of a compiler who has exercised his own independent judgment on the labours of his predecessors, and who hopes that in some instances he has availed himself of what is valuable in their works, while rejecting what is irrelevant or erroneous. And here I think it best to state briefly the view which I have taken of the nature of the Science of Logic, since no compilation can be successful which does not proceed on fixed principles, both as to what it includes and what it excludes. But it does not follow that what I have excluded is in my opinion either useless or impracticable. Logic is not a compendium of all sciences, and what is outside it may be even more valuable than what it contains.

Logic then is the Science of Inference or Proof, meaning by Inference conclusive or indisputable inference. It is therefore limited to what writers term Deduction, to the exclusion of Induction, for in my opinion no inductive inference (subject to the qualifications to be mentioned hereafter) possesses this conclusive and indisputable character. All our experience

of the mortality of mankind does not (conclusively) refute the narrative of the translation of Elijah into heaven. *Argumentum a particulari ad universale non valet* is the watchword of (Deductive) Logic; and no matter how numerous and well-selected the individual instances may be that go to the making up of the Particular (or rather Collective) Proposition, the only conclusive inference is to *some* not to *all.** Conclusions arrived at by induction are only Probabilities, though sometimes Probabilities of so high an order that for all practical purposes we treat them as certainties. If this were not the case, indeed, we should find it impossible to get Premisses for our Syllogisms: but it must be borne in mind that in the following pages I do not intend to lay down any of the *Premisses* employed as perfectly certain and indisputable. I have simply *assumed* certain Premisses to be true, and shown what can be conclusively inferred from them *on that assumption;* but the assumption itself may be incorrect notwithstanding, in which case all inferences from it would fall along with it. Logicians who have endeavoured to obtain indisputably true Premisses for their examples have seldom succeeded in the task, while they have rendered their works uninteresting, and have sometimes led their

* It is to be regretted that some writers insist on using the word "particular" in the sense of "singular" or "individual." Singular propositions are in most respects similar in their logical properties to universals, not to particulars. Nevertheless, no number of singular propositions will conclusively establish an universal proposition, for reasons which will be explained hereafter.

readers to believe that the rules of Logic are inapplicable to any reasonings of real importance. Logic, however, only deals with the sequence of one Proposition from others, and is equally applicable however those others may have been arrived at, and whether they are certain or only probable.

Inferences can only be drawn from one or more Propositions, and the inferences themselves are always Propositions. Hence it is absolutely necessary for a Logician to consider the meaning or import of a Proposition. Indeed it will be seen hereafter that before we can lay down rules for drawing or testing inferences, it is necessary to reduce all Propositions to a fixed number of stated forms. Every proposition too must contain at least two terms and a copula, and therefore Terms must also be dealt with in a Treatise on Logic. Terms usually stand for Things, and they are meaningless to us unless we have some Idea or Notion of the Things they stand for. Hence the Logician cannot pass entirely over either Propositions, Words, Ideas, or Things. But Logic concerns itself with these matters only so far as it is requisite to do so in order to understand thoroughly what it is that we have to draw inferences from, and what it is that we have inferred from it. Beyond this point the structure of Propositions may be left to Grammarians or Rhetoricians; the study of Words may be referred to Philologists; that of Ideas to writers on Psychology; and that of Things to the investigators of the various Sciences which deal with the different properties of

Things. Logic no doubt is in a certain sense a branch of Psychology. Inference is a mental process, and the laws which regulate the process are laws of the Human Mind. But in Logic we do not treat of these laws in reference to the inferring mind, but in reference to the Premisses employed and the results arrived at; and therefore it would be quite out of place to examine the character of any mental process which is not directly concerned in inference. Some writers on Logic, for example, speak of every General Term as having an Extension, because we can *conceive* or *suppose* individuals corresponding to it, though there may be none such in reality: and it is added that whether there is a real extension or not can only be ascertained by experience, and is therefore out of the province of Logic. But in my opinion it is equally out of the province of Logic to inquire whether we can *conceive* or *suppose* anything corresponding to a General Term or not; and from the purely logical point of view all that can be said of any General Term is that it may or may not have an extension real or imaginary. This places in a stronger light the distinction between Extension and Comprehension insisted on in the text. Equally out of place would it be for the Logician to inquire into the precise nature of the Idea, or whatever it is, that is in the mind when we employ a General Term. If the Term has a meaning and one which we understand, it is sufficient for his purpose: but some authors appear to forget that the meaning of a name cannot be either the name itself or any other name.

Some of the reasons for specially distinguishing Logic from Arithmetic and Algebra will appear hereafter in connexion with Sir W. Hamilton's theory of the Quantification of the Predicate. These latter Sciences consist almost entirely of inferences; and as number is applicable to all things, we appear to be drawing inferences about anything indifferently in them as well as in Logic. But the fact is that Logic does not *draw* inferences at all, but only lays down rules for drawing them; while to draw inferences is the exclusive business of the Science of Number. The deductions of this latter Science are themselves as amenable to Logical rules as any other deductions; and when the Logician borrows premisses from it he should set them out as explicitly as if they were supplied by Geology. *All* and *Some*, moreover, are not numerical expressions like the x, y, z, of Algebra. *All men* does not mean any or all of a given or definable number or numbers of men, but the perfectly indefinite body of individuals, past, present, and future, each of whom corresponds with the connotation of the term Man: while *some men* is still more indefinite, since "some" in Logic is not exclusive of *all*. The term *quantity* indeed is but partially applicable to a Proposition and its subject. If it were carefully borne in mind that an Extension is a thing which a term may or may not possess, there would be less danger of confusion on this point.

I need only add a few words as to the history of the Science of Logic. How far it was known to

Aristotle's predecessors it is not perhaps possible now to determine. Most of their writings have not come down to us, and there are doubts as to the genuineness of some of those that have. Much of the teaching of Aristotle's predecessors, the Sophists, moreover, was probably merely oral. Aristotle's, at all events, is the first extant treatise on the subject, and its contents do not appear to have been borrowed from any earlier author. Indeed the supposed anticipations (such as that of Archytas of Tarentum, to whom the Categories have been attributed) rather relate to matters improperly introduced into Logic than to the genuine contents of the Science itself. Aristotle, too, was not only the discoverer but the completer of the Science. He erred not by defect but by excess. He introduced into Logic a great deal that is really foreign to it, and thus prevented its true character as well as its scientific completeness from being seen. Little progress appears to have been made by his successors, and except Galen (who is said to have discovered the fourth figure of the Syllogism), no name worth mentioning occurs until the days of the Schoolmen, or mediæval disciples of Aristotle. Their principal error appears to have consisted in holding that Deduction embraced the whole field of man's belief, and that whatever was true could be syllogistically proved. This led them to widen still further the field of Logic, which had been too much extended by Aristotle himself—to introduce syllogistic forms where they could throw no light on the subject, and to neglect Inductions from experience altogether as

not possessing the precision and conclusiveness of the Syllogism. A revival of Science soon followed, but with it came a disposition to substitute the theory of Induction as employed in Physics for the Logic of Aristotle. Neither the Schoolmen nor their successors the Physicists saw that there was room enough for both, and that each should be confined to its proper sphere. The name of Logic was now given to the substituted theory as well as to that which it sought to supersede, and it was also sometimes employed (conformably to its etymology) to denote a treatise on words. The next step—which however was taken by a comparatively small number of writers—was to combine these three rather incongruous subjects into a single treatise under the name of Logic; or where this was not done to introduce into a work on the Aristotelian Logic a general account of the most approved methods of investigation in Physics, or some of those rules of mental discipline which had been recognised as useful by writers on Psychology or Education. The Science thus came to be overlaid with so much that did not properly belong to it (though useful enough perhaps in its proper place), that it rather resembled a heterogeneous collection selected from every department of human knowledge, than a Science distinct from all others, though possibly dependent on them for the materials which it employs. To get rid of these manifold excrescences thus became the principal task of the revivers of Logic who flourished in the last and the present centuries. Among these Kant occupies the first

place. He defined Logic as the Science of the Necessary Laws of the Understanding and the Reason—a definition approaching nearer to that which I have adopted than might appear at first sight, though it would lead to a fuller discussion of the nature of terms and propositions (or rather of the corresponding mental processes) than I have deemed necessary. This definition was an important improvement on Aristotle no less than on the Schoolmen; and in the hands of Kant and his followers Logic took a form at once more definite and more scientific than before. The strong common sense of the late Archbishop Whately led him to conclusions not dissimilar to those of Kant, whose works he does not appear to have been acquainted with; but besides his deficiencies in Psychology and the exaggerated importance which he attached to language, his desire to make his treatise as *useful* (even theologically) as possible has somewhat lessened its value as a scientific Manual. More recent speculations on the subject do not appear to me to have always taken the right direction. The Hamiltonian Quantification of the Predicate I regard as a mistake which injuriously affects the works of those who have adopted it either wholly or in part, and the error is aggravated by the adoption of mathematical forms to express ordinary Propositions. Other treatises have been injured by an attempt to make new discoveries in a field where few if any remain to be made, or to give a more complicated and learned appearance to a system which is in truth simple and easy. Others again are in great part rather comments

on or explanations of the literature of Logic, than attempts to expound Logic itself as a Science. Of Mr. Mill's Logic it may suffice to say, that it treats of two subjects which ought in my opinion to be kept distinct, and one of which does not admit of the same scientific accuracy as the other. The author, moreover, assumes many questionable propositions from Psychology and some from Physics. These propositions he is, of course, bound to defend, as well as to combat the hostile theories of other Schools of Philosophy; and the result is, that his exposition of Logic is often interrupted by a long argument on a disputed point in Metaphysics, which in more than one instance has been plausibly answered by some writer on the opposite side, whose work must also be consulted if the reader wishes to arrive at a final conclusion. A science built on such a foundation can hardly be regarded as the rule and test of all other Sciences: and Logic, to adopt an expression of Kant's, is not enlarged but disfigured by introducing such discussions into it.

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SUPPLEMENTAL CHAPTERS.

AN INTRODUCTION TO LOGIC.

CHAPTER I.

FORMS OF PROPOSITIONS.

Logic is perhaps the most unpopular of all the sciences. School-boys never think of learning it, and students at college generally content themselves with knowing enough for a pass—a knowledge productive of little practical advantage. Not one man in fifty deems it worth his while to become really acquainted with it, and, when he does, the effects are not always such as to commend the study to others. He is often disposed to argue on all subjects, to broach fallacies and paradoxes, and to look out for supposed logical defects (often arising from mere inaccuracy of expression) in an opponent's argument, rather than to combat it on the merits. Yet if we recollect that Logic is the science which teaches us what inferences we can make from the knowledge we already possess, and how to discern between good and bad reasoning in any speech or book that comes in our way, it will be seen that there are few branches of knowledge which might prove of more prac-

tical utility if properly studied. The unpopularity of the science too, perhaps, is not owing to the nature of the science itself so much as to the manner in which it has been treated of by writers on the subject. The mode of treatment by the Aristotelians has been dry and uninteresting, while at the same time (in my opinion) cumbering the science with needless and useless additions: and of many modern writers it would be more correct to say that they have written on a different subject than that they have dealt with the same subject in a better way. In this chapter and its successors I shall endeavour to explain what I regard as the fundamental principles of the science, and the reader will then be able to judge for himself whether it is worth studying further.

It is unavoidable for a writer on Logic to confine himself to the expression of thought in words, and therefore I shall not inquire how far it is possible to think or reason without words.* No word is admissible in reasoning (or indeed in any kind of composition) which has not a meaning, and the meaning of a word I shall, for shortness, term an **idea**. Whether the idea can exist without

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* Words, however, are generally and, I think, correctly described as the expression of thought, and thought must exist before it can be expressed. Language I believe was invented not to enable men to think, but to enable them to communicate their thoughts to each other. It may be true in one sense that Language is that which renders man superior to other animals; but such Language does not consist in the mere power of producing articulate sounds, which is possessed in an almost equal degree by the Parrot.

the word is a question with which the Logician need not concern himself. In order, however, to be able to reduce all assertions that can be made to a small number of Forms, we find it convenient in Logic to recognise certain parts of speech only—namely, nouns substantive, the verb *to* with its various tenses, and a few particles, such as "not," "if," "either," "or," together with the words therefore," "all" and "some." A pronoun we treat as a noun; an adjective is regarded as part of a substantive (or, as it is called, a **term**), and verbs are regarded as made up of a substantive and the proper part of the verb *to be*. Thus, for example, if any one asserts that *Every horned animal ruminates* (or chews the cud), we make *horned-animal* one term, and instead of the word *ruminates* we write *is a ruminant.* The advantage of this kind of reduction is as follows:—We can often prove something or other to be true of all statements, or assertions or **propositions** (as we call them in Logic) which can be expressed in the form *B is C.* Now *Every horned animal is a ruminant* is of this form, while *Every horned animal ruminates* is not; and hence, though the two propositions have precisely the same meaning, the former is better suited for the purposes of Logic. Again, suppose the proposition had been Every animal with horns on the skull ruminates, we could write this in the form Every animal-with-horns-on-the-skull is a ruminant, which is of the same form as Every B is C. Sometimes a great number of words will enter into a single term in Logic. Thus, in the proposition The removal of the electoral disabilities of women is a measure

advocated by the advanced Liberal party, we must make a single term out of "The-removal-of-the-electoral-disabilities-of-women," while "measure-advocated-by-the-advanced-Liberal-party" is likewise a single term: and by this treatment the above proposition takes the form B is C. That it is sometimes troublesome to put the propositions or assertions we meet with in ordinary life into the forms recognised by Logic will surprise no one who has learned Algebra. Instead of asking the pupil to solve the equation $x^2 - 5x + 6 = 0$, the teacher often begins by saying that A man went to the fair and bought five sheep, &c., &c., in which question a great part of the difficulty consists in reducing the statement to one of the recognised forms of Algebraic equations.

Terms or Nouns Substantive—the only terms admitted in Logic—are either **general** or **singular**.* Man, for example, is a general name because it applies or might apply to more than one individual: so are Member-of-parliament, Emperor-of-Rome, and many other expressions consisting of several words. But John Thompson (or any other proper name) is a singular term, because it stands only for an individual,†

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* Collective terms may be classed as singular for reasons which will be explained further on. The division of terms into general and singular is therefore sufficient for the purposes of Logic.

† It will probably occur to the reader that there might be several individuals called John Thompson, and so John Thompson would be a general name. I am not at present, however, giving accurate definitions, and the difficulty will be removed as he reads on. The *idea* for which John Thompson stands is different in each case, and therefore it is rather a number of singular names alike in sound than a general name.

and so is The-First-Emperor-of-Rome, The-present-Member-for-Dundalk. The meaning of a general term is called a general idea, and that of a singular term might be called a singular idea. Logic is mainly concerned with general terms and general ideas, as it will be found that very little information can be conveyed without introducing them. Thus, if I say *Socrates is wise* (which for logical purposes we express *Socrates is a wise man*) the term *wise* is a general term, because many other persons have been wise as well as Socrates. Indeed general terms (or singular terms made up of general terms limiting and qualifying each other) are the only terms that, properly speaking, have a meaning. If I say of an individual before me that he is John Smith, I merely assert that he is called by that name, but if I say that he is wise or good, learned or stupid, rich or poor, &c., I make an assertion with a real meaning which gives the hearer some information beyond the mere name. Now, according to Logicians, all propositions can be reduced to one or other of the following three forms:—

$$\left.\begin{array}{c}\text{B is C}\\ \textit{or}\\ \text{B is not C.}\end{array}\right\} \qquad (1)$$

$$\text{If B is C, D is F.} \qquad (2)$$

$$\text{Either B is C or D is F.} \qquad (3)$$

Of these (1) is called the **Categorical**, (2) the **Hypothetical**, and (3) the **Disjunctive**. The two latter are made up of two Categoricals (but the Disjunctive

may contain more than two) joined by certain particles; and hence the Categorical must be treated of in the first place. As the meaning of a word is called an Idea, the meaning of a proposition is called a **Judgment.** Hence we often speak of Categorical, Hypothetical, and Disjunctive Judgments instead of Categorical, Hypothetical, and Disjunctive Propositions. But we find it necessary to increase the number of the forms of Categorical Judgments. Our Proposition B is C may mean either *All* B is C or only *Some* B is C: and likewise B is not C may mean either *No* B is C (*i. e.* Every B is not C) or only *Some* B is not C. We must then state which we mean in order to be clearly understood, and thus we obtain four forms of Categorical Judgments or Propositions, viz:

All B is C. (1)
Some B is C. (2)
No B is C. (3)
Some B is not C. (4)

The first of these is often written *Every* B is C, which expresses its meaning more clearly, and the second might perhaps be better written Some Bs are Cs. Thus we would say Some animals are bipeds, rather than Some animal is a biped.* Of these four propositions the first, which is called an **Universal Affirmative,** is usually denoted by the letter **A**: the second, which is called a **Particular Affirmative,** by the letter **I**: the third, which

* If we used the latter form we should probably be understood not as speaking of *several* animals, but of a *single* animal.

is called an **Universal Negative**, by the letter **E**: the fourth, which is called a **Particular Negative**, by the letter **O**. Thus the following argument:—

No B is C.
Some C is D.
Therefore Some D is not B,

would be expressed by the letters EIO. Now Logicians maintain that all Categorical Propositions can be reduced to one of these four forms; for they must either affirm or deny, and the affirmation or denial must either embrace the whole subject spoken about or a part of it only. If the assertion is made about a singular term it is regarded as universal, for I cannot make any assertion about a part of an individual without taking in the whole. *Some* John Thompson would in fact be nonsense: at least unless we were treating the name as applying not to one individual but to several. Logicians also maintain that all statements or assertions whatsoever can be reduced either to one of these four Categorical forms (which we have called A, E, I and O), or to the combinations of them which occur in the corresponding Hypotheticals and Disjunctives. Here experience is our only test. If you can make any statement which cannot be reduced to some one or other of these forms, the science is incomplete, though it may still be correct with regard to every assertion that *can* be reduced to these forms. However, I do not think you will find any exception, and I will take it for granted in the following exposition that none is to be found. Now if we want to discover whether any given argument is valid or invalid, the first

thing we have to do is to break it up into Propositions and reduce these Propositions to some of the forms I have mentioned; and when that is done Logic gives rules for determining whether the argument is good or bad. For instance, it lays down that the form EIO, which I have already given, is always valid provided there are no more than three terms employed in it. But the general rules for testing arguments must be deferred for another chapter.

CHAPTER II.

COMPREHENSION AND EXTENSION, AND ANALYTICAL AND SYNTHETICAL PROPOSITIONS.

General names, as already remarked, have a meaning, and this meaning can usually be resolved into parts. Thus, the name Triangle means a figure bounded by three lines, the name Parallelogram a figure bounded by four right lines, consisting of two pairs of parallels, and the name Man means an animal with two legs, no tail, and certain other peculiarities of external form which I need not enumerate. The meaning of a general name or general term is called by Logicians its *Comprehension* or *Connotation*, or, occasionally, its *Intension*. The different parts of its meaning being usually called *Attributes*, the Comprehension, Connotation, or meaning of a name is frequently said to be a collection of Attributes. The terms Comprehension and Intension are

often applied to Ideas as well as to words, but in this case we must not be misled by the phraseology. The Comprehension or Intension of an Idea is merely the Idea itself: though when the terms are employed it is usually implied that the Idea is a complex one which is divisible into parts, and when we speak of the Comprehension or Intension of the Idea these parts are supposed to be fully enumerated.* A proposition in which the Comprehension of a name is set out at length in this manner is called a **Definition**. Thus, A Triangle is a figure bounded by three lines, is a Definition of the word Triangle. A definition ought to express all the parts of the meaning of the word defined, and nothing more; and as the word Triangle is applied to a figure formed by three lines drawn on a sphere (which is termed a spherical triangle) it would not be a good definition to say A Triangle is a figure bounded by three *right* lines. That would be the definition not of a *Triangle* but of a *Rectilineal-Triangle*. The definition of a word is thus merely its meaning stated at length, and hence its comprehension, its intension, its connotation, and its definition are all different names for the same thing.

The Denotation or Extension of a term is the different individuals to whom it is applicable. Thus, the Denotation or Extension of the term Man is Julius Cæsar, the Duke of Wellington, John Smith, and all the other indi-

* It is more convenient, however, to use the terms Connotation, Comprehension, or Intension for the meaning of a word whether resolvable into parts or not. I accordingly use them in this sense.

vidual men that ever have been or ever will be in the world. It is a matter of accident whether a general name will have any extension or not. Unicorn, Griffin, and Dragon, are general names because they have a meaning, and we can suppose another world in which such beings exist; but the terms have no extension, because there are no such animals in this world. [illegible] Logicians speak of these terms as having an extension, because we can *suppose* individuals corresponding to them. In this way every general term would have an extension which might be either real or imaginary. It is, however, more convenient to use the word Extension for a real extension (past, present, or future) only. If, therefore, we speak of the comprehension and extension of an idea in one breath we must not imagine that the two stand upon the same footing. Every idea (at least unless it is one which is not divisible into parts) must have a comprehension, for the comprehension is the idea itself; but the extension is a thing which the idea may or may not have, and if we want to discover whether it has an extension or not, it is not sufficient to reflect upon the idea in our minds, but we must open our eyes and look about us. The word Dog has a meaning or comprehension. It means an animal with four legs and a tail, and other peculiarities of shape, &c. But if I want to find out whether it has an extension or not, no reflection on the idea of a dog will tell me that. I walk down the next street and I meet a dog. That is the proof that the word has an extension. If I met an Unicorn, that term would have an extension also.

These terms give us a short way of expressing what is asserted in any proposition or judgment. The Proposition Every Man is Rational, asserts that the comprehension or connotation of the term Rational belongs to all the individuals comprised in the extension or denotation of the term Man. Every Equilateral Triangle is Equiangular asserts that the comprehension or connotation of the term Equiangular is found in all the individuals comprised in the extension of the term Equilateral-Triangle. But the extension of any term is determined by its comprehension. The extension of the term Man is simply the whole collection of individuals, present, past, and future, in which the comprehension of that term exists or will exist. The extension of the term Equilateral-Triangle is simply the whole collection of individuals, past, present, and future, in which the comprehension of that term will be discovered. Any individual in whom the whole comprehension of the term Man is not found is not a man. Any individual in which the whole comprehension of the term Equilateral-Triangle is not found is not an equilateral triangle. A proposition, therefore, always asserts that the comprehensions of two or more terms are or are not found together, that is in the same objects. Thus, All Men are Mortal asserts that wherever the comprehension of the term Man is found, the comprehension of the term Mortal will be found along with it. Some Men are Black asserts that the comprehension of the term Man is sometimes found along with the comprehension of the term Black. No Men are Ruminants asserts that the comprehension of the term Man is never

found in the same object with the comprehension of the term Ruminant, and so in all other cases.

This remark leads us to distinguish between two kinds of judgments or propositions which Kant has called Analytical and Synthetical. If the comprehension of one word *includes* that of another, w can always assert with truth that the comprehensior of the latter will be found along with the comprehension of the former. Thus, the term Man means an Animal of a certain kind. The comprehension of the word Man, therefore, includes the comprehension of the term Animal, and we can assert with truth that the latter comprehension will always be found along with the former, or that *Every Man is an Animal.* Every affirmative proposition in which the meaning of the predicate (*i.e.*, the latter term when the proposition is thrown into the form B is C) is a part of the meaning of the subject (or the first term when the proposition is thrown into the form B is C) is true,* but it never conveys any information to the hearer. *The unicorn has a horn* is a true proposition. It explains a part of the meaning of the term Unicorn; but whether the information which it conveys is of any value the reader can judge for himself. These propositions in which the meaning of the predicate is a part or the whole of the meaning of the subject are called **Analytical Propositions or Judgments.** But there is another kind of Proposition

* Of course it is equally true if the predicate expresses the *whole* meaning of the subject whether in one word or several. This is the case with a definition, as already remarked.

in which the meaning of the predicate is not included in the meaning of the subject. Such propositions may be either true or false, and, if true, they always convey real information when we hear them for the first time. Thus, *Every animal with horns on the skull is a ruminant* conveys information to anyone who has not previously observed the habits of cattle, for the idea of an animal with horns on the skull is quite distinct from the idea of an animal that ruminates. That every animal that possesses the one characteristic possesses the other is a fact which could not have been learned by any study of the meaning of words, or any examination of ideas in our minds, but only by actual observation. So that *The earth revolves on its axis* could not be gathered from the meaning of the term *earth*, but had to be inferred from observation of the heavenly bodies, and until recent times it was believed to be untrue. Propositions or Judgments of this kind are called **Synthetical Propositions or Judgments.** They, in fact, contain all our real knowledge. If we had nothing but analytical judgments, the only book that mankind would have need of would be a good dictionary; but in such a state no art or science could exist. These are, therefore, two kinds of Propositions, and the synthetical are of vastly greater importance than the analytical.*

* These two kinds of Propositions have also been distinguished as *Explicative and Ampliative*, and as *Verbal* and *Real*.

CHAPTER III.

IMMEDIATE INFERENCES, CONVERSION, OBVERSION, AND OPPOSITION.

I HAVE already given the four forms of Categorical Propositions which are denoted by the first four vowels in the Alphabet. They are—

All B is C—denoted by the letter		A.
Some B is C—	,,	I.
No B is C—	,,	E.
Some B is not C—	,,	O.

It frequently happens, however, that a writer or speaker avoids telling you distinctly whether he is speaking of All or Some. Thus when a man talks about The human race, The Irish people, &c., it is often very difficult to ascertain which he means, and occasionally, perhaps, he does not know himself.* In dealing with such statements when the reasoner's argument requires that he should have meant All it is better to give him credit for having meant it: if not we should assume that he only meant Some. If a man says B is C, he *must* mean that Some B is C, but he need not mean that All B is so.

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* These propositions in which the quantity of the Subject is unexpressed are called **Indefinite.** In Singular propositions (propositions in which the subject is singular) the quantity is usually left unexpressed, because the whole *must* be meant, as already pointed out. By a kind of poetical or rhetorical licence, however, general terms are often treated as singulars (as in the case of The Human Race, &c.), and the quantity is omitted accordingly. Other instances of indefinites, however, are not unfrequent, *e. g.*, If *men* are wise they will consult their own interests.

Hence you are safe in assuming that he meant the one, but not that he meant the other. I should here mention that *Some* and *All* are not intended to be exclusive of each other. If I say Some bodies are heavy, I do not mean that the other bodies are not heavy; and in general in testing an argument you will do well to attend to what has actually been said, not what might have been said with truth.*

Logicians divide Categorical Propositions into three elements—a **subject**, a **predicate**, and a **copula**. The Copula must always be *is* or *is not* (under which I include such expressions as *was* or *was not*, *will be* or *will not be*, &c.), the former when the proposition is affirmative and the latter when it is negative. In the four forms which I have given above, the letter B stands for the Subject, and the letter C for the Predicate. The subject, in fact, is that about which an assertion is made, and the predicate is that which is asserted about it. In ordinary language the subject is not always

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*Logicians have often laid down that we are to determine the quantity of Indefinite Propositions by the Matter which, according to them, must be Necessary, Impossible, Possible, or Contingent. Without considering what grounds exist for such a classification, this rule evidently assumes that every speaker knows whether the Matter with which he is dealing is Necessary, Impossible, Possible, or Contingent, and means to state not merely the truth but the whole truth. How should we quantify such a Proposition as Equilateral triangles are *not* equiangular? The Matter being Necessary, this negative Proposition is equally false whether the speaker intended his assertion of *All* or only of *Some*. Moreover, the same knowledge is supposed in the hearer as in the speaker.

written before the predicate as it is in Logic. Thus in *Sweet are the uses of adversity*, the word *sweet* is evidently the predicate, and the Logical way of writing the proposition would be *The uses of adversity are sw*[illegible] (You should gather from the context wheth[illegible] all of the uses were intended.) You will ob[illegible] the words *some* or *all* only occur before the *subj*[illegible] proposition. We never say, All men are *all* r[illegible]onal animals, or All men are *some* rational animals. [illegible] logicians have observed that the predicate has really a quantity though not expressed. If I say All men are mortal I plainly imply that Some mortals are men. There must be exactly as many mortals who are men as there are men who are mortals, and all men are included under this latter head. Some mortals must therefore be men, but it has not of course been asserted that all mortals are so. I have then made an assertion about some mortals but not about all mortals. Again if I say All men are rational animals it is clear that Some rational animals must be men; and though it may be true that All rational animals are so, I have not asserted it. My statement is quite consistent with the existence of rational parrots or rational monkeys. The same observation is true of such an assertion as Some men are black. Here there must be as many black things who are men as there are men who are black things; but there may be innumerable black things that are not men. These considerations have led logicians to lay down as a rule that **the predicate of an affirmative proposition is particular.** They

have likewise laid down another rule that **the predicate of a negative proposition is universal.** For if it be true that No man is a stone, it must be equally true that No stone is a man; since if any stone was a man, *that* man would be a stone, contrary to our original assertion. The same observation has been extended to a particular negative, but it might be more correct to say that in that instance the predicate has no quantity at all. In the proposition, Some men are not black, you will not find any assertion made either about some black things or all black things.* The **quantity** of a proposition means its universality or particularity, which again means the universality or particularity *of its subject.* The propositions A and E are *universal*, because in them we make assertions (affirmative or negative) about the *whole* of the subject, while I and O are *particular*, because we are asserting something of a *part* only, leaving the rest undetermined. The **quality** of a proposition means its affirmative or negative character. You will then understand what is meant by saying that the quantity of a proposition depends on (or is the same as) that of its subject, and that the quantity of the predicate depends on the quality of the proposition. It is sometimes said that the quantity of the subject de-

* Sir W. Hamilton and his followers would say it is asserted that No black things are *some* men. Recollecting, however, that the predicate is sometimes written before the subject, I see in this assertion only an awkward and inverted way of writing Some men are not black (things). *Some men* is the subject and *black* (or black things) the predicate in either mode of writing it.

pends on that of the proposition; but this is putting the cart before the horse.

Before finding what inferences can be drawn from two or more propositions, it is desirable to see whether we can draw any inferences from a single one. Now I have already remarked that from any proposition of the form *All B is C* we can draw another of the form *Some C is B.* As many Cs, in fact, must be Bs as there are Bs which are Cs; and the former proposition tells us that all the Bs are so. This kind of inference is called **Conversion.** It will be seen that in it the term which was the original subject has now become the predicate, and *vice versa:* so that *Conversion* means *inferring from one proposition another with the terms transposed.* From what has been said of the quantity of the predicate the reader will easily understand the following rules for Conversion laid down by logicians:

A may be converted into I. (1)
I may be converted into I. (2)
E may be converted into E. (3)
O cannot be converted. (4)

In the case of E and I we can get back to our original proposition by again converting the converse (as it is termed); but if we tried this on A we should find that we had dropped a part of our original assertion and only got back to I. Examples may perhaps illustrate this kind of inference better.

Original Proposition (convertend).	Converse.	
All men are mortal.	Some mortals are men.	(1)
Some men are black.	Some black things are men.	(2)
No men are green.	No green things are men.	(3)
Some men are not black.	(No converse.)	(4)

Again, since the particular proposition Some Bs are Cs does not mean Some Bs *only* are Cs, whatever can be asserted of All can be equally asserted of Some. This gives rise to another kind of inference from a single proposition which is called **Subalternation.** It is only applicable to the Universal propositions A and E, and its rules are—

A may be subalternated into I. (1)
E may be subalternated into O. (2)

Examples—

Original Proposition.	Subaltern.	
Every man is mortal.	Some men are mortal.	(1)
No man is a ruminant.	Some men are not ruminants.	(2)

The subaltern has the same subject and predicate with the original proposition, and differs only in having its subject particular instead of universal. No similar inference can be drawn from a particular proposition. Indeed it is self-evident that from any assertion about a part of a thing no perfectly conclusive inference can be drawn as to a different part, still less as to the whole.

Another kind of inference can be drawn from a proposition which is sometimes called **Obversion.** It consists in turning an affirmative proposition into a

negative, or a negative into an affirmative, with the same meaning. If we allow ourselves to coin a term *not-C* or *non-C* corresponding to any positive term C, this can always be done. Thus—

Original Proposition.		Obverse.	
All B is C,	is equivalent to	No B is non-C.	(1)
Some B is C,	,,	Some B is not non-C.	(2)
No B is C,	,,	All B is non-C.	(3)
Some B is not C,	,,	Some B is non-C.	(4)

The general rule is, in affirmatives affix a *non* to the predicate and alter the copula; in negatives transfer the *not* or *non* from the copula to the predicate. By this means A is reduced to E, I to O, E to A, and O to I. We often find the negative words ready to hand without any coining. Thus The Soul is not mortal (which logicians treat as E) is exactly equivalent to The Soul is immortal (which is regarded as A). After obverting (or turning any proposition into the equivalent one of opposite quality) we can of course *convert* the *obverse*. This process is known by logicians as **Conversion by Contraposition.**

There is another peculiar kind of inference from a single proposition in which we argue from its truth not to the truth but to the falsehood of another, or *vice versa*. This takes place in what is called **Contradiction.** Contradictory propositions have the same subject and predicate, but differ both in quantity and quality: and it will be found that of any two such propositions one must always be true and the other false. There can of course be but two pairs

of them, viz.: A and O, and E and I. To take the former as an example—

Every B is C; (1)
Some B is not C— (2)

it is clear that both cannot be true; for the Bs which are asserted not to be Cs in (2) must be included among the Bs which are asserted to be Cs in (1), that assertion being made of *all* the Bs. But neither can both be false, for it is obvious that if it is false that all the Bs *are* Cs, some at least of them are *not* Cs. The same thing is equally evident of E and I, and the reader, if he chooses, can fill in terms for himself and try the experiment. In general, the fundamental principles of Logic can only be established by an appeal to sound common sense. For if we attempt to *prove* them we must *reason;* and if anyone doubts or denies the principles involved in all reasoning, it is impossible to reason him into a belief in them—in other words, to prove them. But I think the reader who will try to realise in his own mind the meaning of the several propositions employed in these inferences will see that the conclusions drawn from them are self-evident—in fact that they only express the same meaning or a part of the same meaning in other words.

Logicians generally include Contradiction along with what they term *Contrariety* and *Sub-contrariety* under the general head of **Opposition.** All these inferences (if we are to call them so) are from, or rather between, proposi-

tions of opposite quality (*i.e.* one affirmative and the other negative), but with the same terms as subject and predicate. In *Contrariety* both are universal, and therefore one must be A and the other E, while in *Sub-contrariety* both are particular, whence they must be I and O. The inferences which can be made in this case are as follows: Two Contraries cannot both be true; therefore if we know one to be true we can conclude that the other is not true. This follows at once from the rules of Contradiction and Subalternation. If A, for example, be true, its subaltern, I, would be true, and the Contradictory of that Subaltern, E, would be false; hence A and E cannot be both true. Two Sub-contraries, on the other hand, cannot both be false; hence if we know one to be false we can infer the truth of the other. This follows in the same way as before. If, for instance, I be false, its Contradictory E is true, and therefore so is O, which is the Subaltern of that Contradictory: hence I and O cannot both be false. The only useful kind of Opposition, however, is Contradiction. Since of two Contradictories one is always true and the other false, it is exactly the same thing to prove a proposition as to disprove its contradictory; and it is likewise the same thing to prove the contradictory as to disprove the proposition. But for the purposes of reasoning it is often more convenient to adopt one course rather than the other. Thus, in the Sixth Proposition of his First Book, Euclid establishes the proposition that if the base angles of a triangle are equal its sides are equal,

by disproving its Contradictory;* and he establishes the Nineteenth as well as several other propositions in the same way. These instances will illustrate the use of Contradictory propositions in reasoning.

CHAPTER IV.

IMMEDIATE INFERENCES.

In my last I enumerated the principal kinds of immediate inference, namely, Conversion, Subalternation, Obversion, and Opposition. All these inferences appeal to common sense, and can hardly be established otherwise; but the reader will perhaps be in a better position to judge of their conclusiveness by seeing a collected list of them drawn out with examples such as we ordinarily meet with. Let us then see what inferences can be drawn in this manner from each of the propositions A, E, I, and O, taken in order.

I. From A, taking as an example *All horned animals are cloven-footed*, we can infer the *truth* of

Some horned animals are cloven-footed—Subaltern ; (1)
Some cloven-footed animals are horned—Converse; (2)
No horned animals are without cloven feet—Obverse ; (3)
No animals without cloven feet are horned—Converse by contraposition; (4)

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* The contradictory is disproved by showing that the assumption of its truth would lead us to *contradict* the Fifth Proposition, which has already been proved. Contradiction is thus employed twice in the argument.

and the *falsity* of

> No horned animals are cloven-footed—Contrary; (5)
> Some horned animals are not cloven-footed—Contradictory; (6)

to which we might add the converse of the contrary.

(I have purposely made one or two slight alterations from the strict logical form in these and the following inferences, in order to show how the wording may be altered without affecting the meaning.)

II. From the proposition E, or *No vertebrate animals are cold-blooded*, we obtain the following inferences:

> Some vertebrate animals are not cold-blooded—Subaltern. (1)
> No cold-blooded animals are vertebrates—Converse. (2)
> All vertebrate animals are warm-blooded—Obverse. (3)
> Some warm-blooded animals are vertebrates—Converse by contraposition. (4)

To which may be added—

> Some cold-blooded animals are not vertebrates—Subaltern of converse; (5)

and we can infer the falsity of

> All vertebrate animals are cold-blooded—Contrary; (6)
> Some vertebrate animals are cold-blooded—Contradictory. (7)

To which might be added the converses of both contrary and contradictory.

III. From the proposition I, or *Some men are black*, we obtain the following inferences:

> Some black things are men—Converse; (1)
> Some men are not non-black—Obverse. (2)

(Here you will observe there is neither subaltern or

converse by contraposition; for we can only subalternate an universal proposition, while I is particular; and the converse by contraposition, being the converse of the obverse proposition, there can be none where the obverse proposition is O, as in this case.)

And we can infer the falsity of

No men are black—Contradictory, (3)

with its converse: but there is here no contrary, as contrariety only exists between universals.

IV. From the proposition O, or *Some men are not white*, we can draw the following inferences:

Some men are non-white—Obverse. (1)
Some non-white things are men—Converse by Contraposition. (2)

(The reader will see why there is no subaltern or converse.)

And we can infer the falsity of

All men are white—Contradictory. (3)

To the inferences as to the falsity of other propositions I might have added the contradictory or contrary of the obverse or of the converse by contraposition in each instance. The reader will notice that sub-contrariety does not occur in this list of inferences, and in fact, assuming the *truth* of any proposition, no inference whatever can be drawn from it by sub-contrariety. It is only if we assume it to be *false* that we can infer the truth of its sub-contrary. And here it might appear that I ought to have added to my Table a list of the deductions which might have been made if the original proposition was known to be false instead of true. But this is unnecessary, for a reason already

mentioned. To assume the falsity of any proposition is exactly the same thing as to assume the truth of its contradictory. If you wish then to know what inferences can be drawn from the falsity of the proposition A, you will find them under the inferences from O regarded as true; and if you wish to find what inferences may be drawn from the falsity of E, look to those deducible from the truth of I. The same rule applies to the falsity of O and of I, which are equivalent to the truth of A and of E respectively. Hence no immediate inference is ever drawn by sub-contrariety, and that kind of opposition might have been wholly omitted—unless indeed to caution you as to what inferences *cannot* be drawn.

For you are to understand that our Table is an *exclusive* Table. You are warranted in drawing the inferences therein mentioned *and no others.* You are not to convert A into A. From *All men are mortal* you cannot infer that *All mortals are men.* Still less of course can you convert I into A; nor can you infer an affirmative converse from a negative proposition (except in obversion or contraposition), or *vice versa.* Again, an assertion may be true of a part without being true of the whole, and consequently there is no process corresponding to subalternation by which you can infer A from I, or E from O. From *Some men are black* you cannot infer that *All men are black;* nor the reverse (or contradictory), for though the latter may be true, it has not been asserted. From *Some men are not black,* in like manner, you cannot infer that *No men are black.* In fact there is a valuable general rule applicable to all these inferences, viz., that you can-

not increase the quantity of a term (unless indeed you are inferring the falsehood instead of the truth of the proposition in which the term occurs with the larger quantity). For it is self-evident that something may be true of a part, but not of the whole or of a different part of the same whole: and hence, whenever a term is particular in the original proposition it cannot be universal in the inference derived from it. For though *Some B is C* does not *mean* Some B *only* is C, it must always be *consistent* with the latter assertion: otherwise we ought to assert All B is C at once—which would thus be a direct assertion, not an inference from Some B is C. This rule, which is expressed in the Latin phrase **Argumentum a particulari ad universale non valet,** is as applicable to deductive reasoning of any length as to immediate inferences; and any conclusion containing an assertion about a whole is invalid if the propositions from which it was drawn (no matter how many of them there may be) contain assertions about a part only. Another rule is, that **if the same terms are preserved, the quality of the proposition cannot be altered.** This, again, is not applicable to cases in which from the truth of one proposition the falsity of another is inferred; and it is not inconsistent with what has been said of obversion and contraposition, because in them one of the terms is altered (C becoming non-C). This rule rests on the principle, that in asserting one thing to be either wholly or partially identical with another we allege nothing as to their differences, and that in asserting them to be wholly or partially different we allege nothing as to their

identity. The reader might here imagine that in alleging a partial identity we implied that there was a difference as to the other part; but in Logic Some Bs are Cs does not mean Some Bs *only* are Cs. It simply says that some Bs are Cs, leaving it quite undecided whether the *other* Bs are or are not Cs. The fact is, that Some Bs *only* are Cs is not a single proposition but two propositions, viz., Some Bs are Cs, and, Some Bs are not Cs; and a similar observation will apply to some other forms which might be regarded at first sight as real additions to the four forms, A, E, I, and O, admitted by logicians. The two foregoing rules are of wide application. The latter, for example, precludes the drawing of a negative conclusion from any number of affirmative propositions.*

There is another rule of still wider application involved in subalternation which is often expressed thus:—*Whatever is true of a whole is true of any part of the whole.* It would be better stated thus: **Whatever is true of every member of a class is true of any member of any part of that class.** For there are some kinds of wholes whose properties do not belong to their parts. You cannot infer that because Ireland is an island the county Londonderry is an island, or that because all armies consist of infantry, cavalry, and artillery, a private soldier consists of in-

* The rule as stated in the text is of course inapplicable to the case in which some of the propositions from which the conclusion is inferred are affirmative, and others (or rather another) negative. In such cases the conclusion must differ in quality from some of the propositions from which it is inferred, and it will be seen hereafter that it will always be negative.

fantry, cavalry, and artillery. I have altered the wording of the axiom to exclude this kind of wholes and parts, and also to call your attention to a fact which it is very important to bear in mind in logical reasoning, viz., that **in all the four propositions A, E, I, O, the subjects must be taken distributively, not collectively.** All men are rational does not mean that all men taken collectively possess the attribute of reason, but *that each individual man* does so. Some men are white does not mean that some men taken collectively are white (which would be true if these men were divisible into sets of two whose colours were those known as complementary), but that *each one of some men* is a white man. The language of logicians sometimes tends to conceal this latter fact: for not only are the terms *All* and *Some* ambiguous (as respects their distributive or collective use), but a term taken universally is often spoken of as *distributed*, and a term taken particularly as *undistributed.* It is therefore necessary to remind you that in particular no less than in universal propositions the subject is taken distributively, not collectively. Whenever a term has to be used collectively it may be treated as a singular term. The British Army won the battle of Waterloo is, from the logician's point of view, a singular proposition, no less than The Duke of Wellington won the battle of Waterloo; The Parliament meets at Westminster is as much a singular proposition as The Queen resides at Balmoral. There is therefore no necessity for introducing new forms of propositions to meet the case of collective terms. A, E, I, and O are sufficient for all our needs; but if care be not taken we

shall sometimes fall into what is called a fallacy of composition or division. Thus, if after enumerating the primary colours it was stated that All these colours make white, I should fall into the fallacy of division if I were to infer that Some of these colours make white; for the former proposition has not been asserted of *every* colour, but only of all the colours collectively. This caution, too, will prevent you from confounding logical with algebraic reasoning, which at first sight are very much alike. Thus, Some B is C would be treated by some writers, even on Logic, as an *equation*, viz., Some B = Some C, and the conversion of this proposition would consist in merely writing down the same equation with its terms transposed, viz., Some C = Some B. In this reasoning it is forgotten that in Algebra the terms are always used collectively, but in Logic distributively. All armies = all soldiers, is a perfectly correct algebraic equation, but as no army is a soldier it is inadmissible in Logic, except, indeed, as a singular proposition of the same kind, as Hyde was Clarendon. I mentioned before that logicians treated such singulars as universals; but they cannot be dealt with as such in all respects. You cannot employ the terms *Some* B or *Some* C where B or C are proper names. To infer from Hyde was Clarendon that *Some* Hyde was Clarendon*

* Unless by Some Hyde we mean A certain Hyde; in which case the word "Some" is in fact superfluous, and the meaning is exactly the same as before. This, too, arises from an ambiguity in the English word *Some.* In Latin, for example, we should have to use the word *quidam* instead of *aliquis*, which would make the change of meaning immediately evident.

would be nonsense. Hence, with such singulars there is no room for subalternation or contradiction, and the converse and contrary of a singular also possess certain peculiarities—the latter in particular obeying the laws of contradiction; for though Hyde was Clarendon and Hyde was not Clarendon would both be deemed universal propositions (A and E) in Logic, one must be false and the other true. It would occupy too much time to examine these peculiarities in detail, and they will be different when the subject or predicate only is singular, and when both of them are so. I only mention them here because all the same peculiarities belong to collective propositions. From All soldiers are all armies, you cannot infer Some soldiers are armies—still less This soldier is an army.

In stating that the above list of immediate inferences is exclusive, I only mean that it is so, provided the same terms are preserved, or at least that the original subject is preserved and the original predicate only altered by the addition of the *non* or *not*. If we allow ourselves to alter the terms in any way we choose, many other immediate inferences may be drawn: for example, from John is the *father* of Thomas I may conclude that Thomas is the *son* of John. In fact from this point of view any proposition which expresses the same meaning, or part of the same meaning with the original one, may be regarded as an immediate inference from it, and the modes of expressing the same meaning in different words are often very numerous. No general rules, however, can be laid down for drawing such inferences.

They depend chiefly on accidents of language, and in particular on the number of synonymous words, which a dictionary would give more assistance in tracing than a treatise on Logic. Such reasonings, however, can always be expressed as arguments from two or more propositions, by supplying an additional proposition whose truth (though it may be self-evident) is essential to their validity. Thus, in the instance I have given, we might supply the hypothetical proposition If John is the father of Thomas, Thomas is the son of John, and thus form what is called a hypothetical syllogism. For logical purposes this is the best way of treating all such inferences. It shows exactly what is requisite to make them valid. If the supplied premiss is a self-evident proposition, as in the instance just given, the inference is valid; but if the proposition so supplied be not self-evident, the reasoning is insufficient and the inference is not correctly drawn. For the purpose of testing arguments, therefore, it is better to regard no immediate inferences as valid except those which I have enumerated. In general it is a useful rule to require a reasoner to *state* every step in his argument; and if he does not state it himself you can make use of the rules of Logic to state it for him. In such cases you will very often find that the weak point of his argument does not lie in what he has said, but in what he has avoided saying, though it is equally essential to his case.

CHAPTER V.

THE SYLLOGISM AND ITS RULES.

HAVING stated the inferences which may be drawn from a single categorical proposition, we now come to those which can be drawn from two. I must here again remind you of what is meant by the comprehension and the extension of a term. The comprehension of a term is the collection of attributes which it signifies, *e.g.* the comprehension of the term man is animality, reason and a certain external form. Its extension is the number of individuals which possess these attributes, *e.g.* in the case of man, Julius Cæsar, Tom Brown, and every one else whom you choose to name. Now an inference from two propositions is called a **syllogism**, and when the two propositions are categorical, the syllogism is called a **categorical syllogism**. All categorical syllogisms depend on the following axioms:—

1. **If two terms coincide, as to their extensions, with the same third term, or with the same part of the same third term, their extensions coincide with each other.**

2. **If one of them coincides, as to its extension, with a third term, and the other does not coincide with it, the extensions of the two terms do not coincide.**

To explain. When I say *All men are animals*, I say that the whole extension of the term man coincides with the extension of the term animal: but not that the whole extension of the term animal coincides with that

of the term man. (You will recollect that the predicate of an affirmative proposition is particular.) Again, if I say *No men are cold-blooded,* I say that no part of the extension of the term man coincides with any part of the extension of the term cold-blooded. Thus generally an affirmative proposition states that the extensions of two terms coincide either in whole or in part, and a negative proposition asserts that they do not coincide either in any part (universal negative) or in some part (particular negative). Taking the four propositions A, E, I, O in order;

1. *All B is C* asserts that the *whole* extension of B coincides with (or is included in) the extension of C (but not *vice versa*).

2. *Some B is C* asserts that the extension of B coincides in part with that of C.

3. *No B is C* asserts that no part of the extension of B coincides with any part of the extension of C.

4. *Some B is not C* asserts that a part of the extension B does not coincide with any part of the extension of C.

Now when we compare the extension of our two terms with that of one and the same third, in order to ascertain whether they do or do not coincide with each other, the two terms are called the **extremes**, and the third, with which we compare them, is called the **middle term** The two extremes afterwards receive names of their own. That which becomes *the predicate of the conclusion* is called the **major term**, and that which becomes *the subject of the conclusion* is called the **minor term.**

It is evident from the foregoing Axioms that in every

syllogism we must assume two propositions. We have in fact to compare the extension of the middle term with that of each of the extremes successively, in order to see whether it coincides or does not coincide with each of them. The two propositions thus assumed are called **premisses.** The one in which the extension of the middle term is compared with that of the major term is called the **major premiss,** while that in which the extension of the middle term is compared with the extension of the minor term is called the **minor premiss.** Finally, the proposition which is inferred from these two is called the **conclusion.** Every syllogism then consists of a major premiss, a minor premiss, and a conclusion. In the first, the major term is compared with the middle, to see whether they do or do not coincide as to their extensions: in the second, the minor term is compared with the middle, to see whether they do or do not coincide as to their extensions; and then the conclusion infers that the extensions of the major and minor terms do or do not coincide with each other, according as the premisses conform to the first or the second of the Axioms already laid down. Hence it follows that *in every categorical syllogism there are three terms only,* namely, the major, the minor, and the middle. *There are likewise three propositions only,* namely, the major premiss, the minor premiss, and the conclusion.* In Logic the major premiss is

* In Logical treatises the words Major and Minor are sometimes used for the Major and Minor *Terms,* and sometimes for the Major and Minor *Premisses.* The context will always show which is meant, and the student should take care to ascertain this before going farther.

usually written first, but the order is really a matter of indifference. The true distinction is this: The major premiss is that whose terms are the middle term and the predicate of the conclusion: the minor premiss is that whose terms are the middle term and the subject of the conclusion.

I now proceed to determine what *rules* can be deduced from our two *axioms*.

First, then, *the middle term cannot be taken twice particularly*. For if it were taken particularly in both premisses we should first have compared the extension of the major term with a part of the extension of the middle, and then compared the extension of the minor term with a part of the extension of the middle. Now it is here possible that we might have compared them with different parts of the extension of the middle, in which case no conclusion could be drawn.* Take for example the following argument:—

Every man is an animal—Major Premiss.
Every horse is an animal—Minor Premiss.
∴ Every horse is a man—Conclusion.

The conclusion here does not follow; for the term

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* This is always the case if the parts with which we have compared the two extremes are *undefined* parts, and it is with such undefined parts that we have to deal in Logic—with Subjects quantified by the word *Some* and Predicates of Affirmative Propositions. No doubt if we were dealing with *defined* parts, each of them might be such a fraction of the whole as to show that they embraced something in common. Even this,

man only coincides as to its extension with a part of the extension of the term animal, and the term horse likewise only coincides as to its extension with a part (and a different part) of the extension of the term animal. Hence the terms man and horse have not been shown to coincide (as regards their extension) with the *same* third term, and therefore we can infer nothing as to their coincidence or non-coincidence with each other. The argument is simply worthless. But it would be otherwise if we had compared the extension of one extreme with the whole, and that of the other extreme with a part of the extension of the third term. For the whole includes every part; and if in one premiss we compared one extreme with the whole, and in the other premiss we compared the other extreme with a part, it is plain that we should have compared both extremes with the same part. Thus, the following is a valid syllogism :—

All men are bipeds.
All bipeds are warm-blooded.
∴ All men are warm-blooded.

Here, though man only coincides with biped in a part of its extension, yet as warm-blooded coincides with biped as to its whole extension, it must coincide with

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however, would be proved by Arithmetic, not Logic, and the argument could not be regarded as *logically* complete without introducing some proposition relating to fractions borrowed from Arithmetic. The reasoning would thus be extended to at least *two* Syllogisms, and there would no longer be a *single* Middle Term to which the above rule could be applied.

it in the same part that man coincides with: and as the two terms, man and warm-blooded, coincide as regards their extension with the same part of the same third (biped), they must coincide with each other. A syllogism in which the middle term is taken twice particularly is said to have an **undistributed middle.** Accordingly it follows immediately from our axioms that a syllogism with an undistributed middle is invalid. This is our **first rule. The middle term must be taken universally once at least in the premisses.**

Again, if the extension of both extremes coincide, whether in whole or in part, with that of one and the same middle, the conclusion is plainly that they coincide with each other, not the reverse; and even if their extensions are shown to coincide with different parts* of the same middle we cannot thence infer their non-coincidence with each other. Hence follows a **second rule, viz., From two affirmative premisses a negative conclusion cannot follow.** There may be an affirmative conclusion or there may be no conclusion (the latter being the case if there is an undistributed middle), but there cannot be a negative conclusion.

Again, if the extension of one of the extremes coincides with that of the middle (whether wholly or in part), and that of the other extreme does not coincide with it, we can infer nothing as to the coincidence of the two extremes. There may be a conclusion asserting

* Or rather with parts not *shown* or *stated* to be the same, and which therefore *may* be different. This is what occurs in the cases referred to.

a non-coincidence, or there may be no conclusion, as will be the case if the middle term is undistributed; but a conclusion asserting coincidence cannot follow according to our axioms. Hence follows **a third rule. If either premiss be a negative, there cannot be an affirmative conclusion.**

A *fourth rule* depends on the principle already laid down that an argument *a particulari ad universale* is invalid. Hence, whether our premisses assert a coincidence or a non-coincidence of extensions, if we have spoken of a part only of the extension of either of the extremes in the premisses, our conclusion cannot contain an assertion about the *whole* extension of that extreme. This indeed is almost self-evident; for it seems manifest that the part of the extension of the extreme about which we have made no assertion may either agree or disagree with the other part of which alone we have said anything.* **Hence, No extreme which is particular in the premisses can be universal in the conclusion.** This fault is known by the name of illicit **process.** It is an illicit process of the major if the major term is particular in its premiss and universal in the conclusion, and an illicit process of the minor if the same thing happens to the minor term. The **fourth rule**

* The reader must, however, bear in mind that a particular proposition is not limited to some *only* of the objects denoted by the subject-term. It merely abstains from making any assertion about *all*. It is indeed more accurate to say that there *may be* another part about which no assertion has been made than that there *is* one.

may therefore be expressed thus: **No syllogism is valid if an illicit process of either extreme occurs in it.**

A fifth rule is, that from two negative premisses nothing follows. For here neither of the extremes coincides in its extension with the middle, and in this case neither of our axioms (nor any other axiom that could be invented) tells us whether they agree or disagree with each other. Hence, in this case we can draw no inference whatever.

From these five rules we can draw some useful consequences. First, there is always the same number of universal terms in the predicates of the premisses as in the predicate of the conclusion (which must be either 1 or 0). For since two negative premisses are inadmissible, the predicates of the premisses cannot be *both* universal or contain *two* universal terms. If they contain *one*, then one premiss will be negative, whence the conclusion must be negative by our third rule and will therefore have an universal predicate; while if the predicates of both premisses be particular (*i.e.* if both premisses be affirmative) the conclusion will also be affirmative, and will therefore have a particular predicate. Secondly, by our first rule the middle term must be once at least universal in the premisses, and it does not enter into the conclusion at all; while by the fourth rule no term can be universal in the conclusion unless it has been so in the premisses. Hence the premisses must contain one more universal term than the conclusion, and as this excess cannot occur in the predicates, it must occur in the subjects. Hence, *if one premiss be*

particular (*i. e.* have a particular subject) *the conclusion* (*if any*) *will be particular.* For from what has been said the premisses must have one more universal subject than the conclusion, and as in this case the premisses have but one universal subject the conclusion can have none—in other words, it must be particular. Further, *from two particular premisses no conclusion can be drawn.* For in every valid syllogism the premisses must have one more universal subject than the conclusion: but if both premisses be particular the premisses do not contain any universal subject, and therefore no conclusion is possible. It is likewise evident that *the premisses can never contain more than two universal terms in excess of the conclusion;* for as there can be no excess as respects the predicates, the greatest possible excess will be attained when the subjects of both premisses are universal and the subject of the conclusion is particular, *i.e.* when there are two universal premisses and a particular conclusion. Conversely, *if the excess of universal terms in the premisses over the conclusion be two, we must have two universal premisses and a particular conclusion.* Some of these deductions are sometimes laid down as additional rules, but it is better to regard them as consequences of the foregoing five. These are the entire number of general rules, and it will be found on inspection that every syllogism which does not violate one or other of them is valid; but this fact is by no means obvious at first sight, and I can only hope that the succeeding chapters will make it apparent.

CHAPTER VI.

THE SYLLOGISM—NUMBER OF VALID MODES.

In speaking of the number of terms in a syllogism there is an ambiguity which, if not cleared up, might render the various rules laid down in the last article difficult to follow. The principle on which the entire theory depends is, that in reasoning two terms are to be compared (as regards their extension) with the *same* third in the premisses, and from thence their mutual relation to each other is to be inferred in the conclusion. Every legitimate syllogism, then, can have but *three* terms, namely, those which we have designated the major, the minor, and the middle. But in another sense the premisses of a syllogism contain *four* terms, each premiss having both a subject and a predicate. The explanation of the ambiguity is, of course, that the middle term occurs *twice* in the premisses, namely once in each of them. The general rules laid down in the last article have gone upon the latter mode of counting the number of terms. Thus if the middle term (which does not occur in the conclusion) was universal in both premisses, we should have spoken of it as an instance in which the excess of universal terms in the premisses over those in the conclusion was two instead of one. The reader will have to bear this ambiguity constantly in mind.

The rules which determine the validity of syllogisms having been laid down, the next step is to determine

the varieties of valid and invalid syllogisms. Syllogisms differ in respect of figure and mode. The **figure** of a syllogism is determined by the position which the middle-term occupies in the premisses. If it is the subject of the major premiss and the predicate of the minor, the figure is called the **first figure**; if the predicate of both premisses, the **second figure**; if the subject of both, the **third figure**; while if it is the predicate of the major and the subject of the minor, it is the **fourth figure.** It is plain that no more than these four orders of terms is possible. The middle term, as we have seen, must occur in both premisses, and when occurring it must be either predicate or subject. Hence, taking the two premisses together, the middle term must be either the predicate of both (2nd figure), the subject of both (3rd figure), or the predicate of one and the subject of the other (including both the 1st and the 4th figures). In this last case it *often* happens that we may treat the premisses as forming a syllogism in the 1st or the 4th figures indifferently, the order of terms in the conclusion being varied in consequence. Thus being given the premisses

All men are bipeds, (1)
Some long-lived animals are men, (2)

we may either treat (1) as the major premiss and (2) as the minor, and draw the conclusion,

Some long-lived animals are bipeds,

in the 1st figure; or else treating (2) as the major,

and (1) as the minor, we may form a syllogism in the 4th figure with the conclusion,

Some bipeds are long-lived animals

(which might also be reached by converting the conclusion of the former syllogism).

This, however, is not invariably true. It will be seen hereafter that in some instances the premisses will give a conclusion in one of these figures but none in the other. For instance the premisses—

Some cloven-footed animals are ruminants, (1)
No men are cloven-footed, (2)

will not warrant any conclusion if we treat (1) as the major premiss and (2) as the minor:* but inverting this order we can draw the conclusion,

Some ruminants are not men,

by a syllogism in the 4th figure. The four figures are therefore really distinct.

The **mode** of a syllogism is a mere name for the three propositions that form it, arranged in their order. In this order, for the sake of uniformity, the major premiss is written *first*—not of course that there is any reason for stating it first in an actual argument. The conclusion (which is similarly written *last*) follows just as

* For the conclusion (if any) would be negative, and therefore the major term would be universal in it. But this term is particular in the major premiss, and the conclusion would thus involve an illicit process of the major, as it has been termed.

much from the premisses stated in one order as in the other, and the force of the reasoning sometimes appears more obvious when the minor premiss is written first. It is often convenient, too, to begin by stating the conclusion, *i.e.* what you are *going* to prove. It is necessary, however, to have some uniform order of writing them in Logic, in order that the various modes should have fixed names. In the names of these modes we make use of the vowels A, E, I, and O, to denote the propositions as already explained. Thus the mode EIO means a syllogism in which the major premiss is E (that is an universal negative), the minor premiss I (that is a particular affirmative), and the conclusion O (that is a particular negative). If we intended to make I the major, and E the minor, the mode would be called IEO. A mode therefore is always designated by three vowels, the first of which stands for the major premiss, the second for the minor premiss, and the third for the conclusion. It is a valid or legitimate mode if the conclusion follows from the premisses, an invalid or illegitimate mode if it does not. Thus in the above example it will be seen hereafter that EIO is a valid mode, and IEO is an invalid one. This may seem puzzling to the reader who has just been told that the conclusion equally follows from the premisses in whatever order they are written. And so it does: but the fact is that the two propositions here called O are not identical. They are the simple converses of each other, and O does not admit of being simply converted. One of these O's follows from the premisses E and I, however written: the other does not follow from them, however written.

Like most of our logical terms, however, the word "*mode*" also admits of an ambiguity. A mode, as I have described it, may be in any figure; and the mode EIO is, in fact, a valid mode in each of the four. But it takes special names according to what figure it is in, and the word mode, so applied, comes to mean a combination of what I have previously called mode *and* figure. When *mode* is used in this latter sense it is no longer designated by three vowels only, but by three vowels in combination with certain consonants the meaning of which will be explained hereafter. Thus the mode EIO when limited to the first figure is called *Ferio* (FErIO), when limited to the second figure it is called *Festino* (FEstInO), when limited to the third figure *Ferison* (FErIsOn), and when limited to the fourth figure *Fresison* (FrEsIsOn.) Each of these four are spoken of as modes and referred to by their names. The valid modes have got names of this latter description, but the invalid modes have not; and hence, in speaking of an invalid mode it is only from the context that you can learn in what sense the word *mode* is used. It is otherwise with a valid mode. If a writer speaks of the mode EIO, it is plain he is using the word mode in the former sense; and if he speaks of the mode *Ferio* he is using it in the latter.

Now we wish to ascertain the total number of valid modes in both senses, but especially in the latter. Writers usually commence this inquiry by ascertaining the total number of possible modes, valid and invalid. Now although the three propositions of a syllogism must be three distinct propositions, there is nothing

to prevent the same vowel standing for two, or even for all three of them. The mode AAA, for example, means a mode in which the three propositions are all universal affirmatives, which they may very well be, and preserve their own distinctness notwithstanding. Thus—

> All animals are mortal.
> All men are animals.
> All men are mortal.

Hence, if we wish to find out the whole number of possible modes (valid and invalid), we must recollect that the major premiss may be any one of the four propositions A, E, I, O: so may the minor premiss, and so may the conclusion. The total number, then, if we use the term mode in the first sense, is $4 \times 4 \times 4 = 64$. But each of these modes may (whether validly or otherwise) be in any one of the four figures, and hence if we wish to find the total number of possible modes in the second sense of the term, we must multiply the previous result by 4 (the number of figures), which gives us 256. The total number of possible modes is therefore either 64 or 256, according to which sense we are using the word *mode* in.

But in ascertaining the number of valid modes, our task will be much simplified if we confine our attention in the first place to the premisses, using the term mode in the first sense before passing on to the second. Now it is evident that the total number of possible pairs of premisses is 16 (any one of the four possible

majors A, E, I, O, being followed by any one of the four possible minors). Let us set out the list:—

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Major	A	A	A	A	E	E	E	E	I	I	I	I	O	O	O	O
Minor	A	E	I	O	A	E	I	O	A	I	E	O	A	E	I	O

Testing these sixteen pairs by the rules laid down in the last chapter we find that the pairs of premisses marked 6, 8, 14, and 16 in the above list are invalid according to the fifth rule, for having two negative premisses. They cannot, therefore, lead to any conclusion in *any* figure. Again, the pairs of premisses marked 10, 12, and 15, have two particular premisses, and therefore, according to one of the deductions given at the end of last chapter, no conclusion can be drawn from them in any figure.

The same thing can be proved, though not so immediately, of the pair of premisses IE (I being the major premiss) marked 11. For since one premiss is negative the conclusion cannot be affirmative (rule 3). If there be any conclusion, therefore, it is negative, and consequently it has an universal predicate, *i.e.* the major term is universal in it. But both terms of I being particular, the major term is particular in the major premiss whether it is the predicate or the subject, and the syllogism (in whatever figure it is constructed) must thus contain the fault which we have termed an illicit process of the major term contrary to the fourth rule.

There remain then eight pairs of premisses. Of these three consist of two affirmative premisses, viz., AA, AI,

and IA, and the other five of one affirmative and one negative premiss, viz., AE, AO, EA, EI, and OA. The first three can only have one of the two affirmative conclusions A or I, while the last five can only have one of the two negative conclusions E or O. But we can restrict the number of valid conclusions still farther. For by one of the corollaries at the end of the last chapter it appeared that if one premiss of a syllogism be particular, the conclusion (if any) must be particular. Hence, from the pairs of premisses AI and IA, the conclusion must be I, and from the pairs AO, EI, and OA the conclusion must be O. From two universal premisses, however, a particular conclusion may be drawn (for, in fact, whenever an universal conclusion can be drawn, the corresponding particular follows at once from it by subalternation; and as the conclusion was drawn from the premisses, whatever can be drawn from *it* may be drawn from *them* also). The pair of premisses AA may therefore have either of the conclusions A or I, and the two pairs AE and EA may have either of the conclusions E or O. We thus obtain a complete list of legitimate or valid modes, using that term in the first sense. They are as follow (adopting the order in our table):—

AAA, AAI, AEE, AEO, AII, AOO, EAE, EAO, EIO, IAI, OAO,

in all eleven. It might be at first imagined that to find the number of legitimate modes in the second sense of the word we should multiply this result by 4, the number of the figures. But several modes which are valid in one figure are invalid in others. For example, in the

second figure the pair of premisses AA is invalid for undistributed middle (rule 1), since in that figure the middle term is the predicate of both premisses, and when both are affirmative it is twice particular.

This investigation must be left for the next chapter. I conclude this chapter by stating what is meant by an **useless** mode. A mode is regarded as useless if a particular conclusion is drawn where the premisses warrant an universal one—or rather an universal one with the same subject and predicate. The conclusion in such a case really follows from the premisses (whereas in the case of an invalid or illegitimate mode it does not follow at all), but, as the same premisses afford a more valuable conclusion, it is considered useless to draw the less valuable one. Thus, to take our former instance—

All animals are mortal;
All men are animals;
All men are mortal.

Here we might have equally concluded *Some men are mortal;* but for the reason already stated it is considered *useless* to do so. But if we draw a particular conclusion from the same premisses, treating the second premiss as the major (and thus forming the syllogism in the fourth figure instead of the first), the mode is not considered useless. The conclusion in this way is, *Some mortals are men;* and as the premisses do not enable us to infer that *All mortals are men,* the mode AAI of the fourth figure is not considered useless, like AAI of the first. Many writers on Logic do not include useless modes in their

list of legitimate modes, and in this way our catalogue of eleven is reduced to ten; for it will be seen hereafter that the mode AEO is useless in the two figures in which alone it is legitimate.

CHAPTER VII.

SPECIAL RULES OF THE SYLLOGISTIC FIGURES.

If we refer back to the general rules of syllogism, and the deductions derived from them, we shall find that some of the faults which they prohibit affect *terms* only, while others affect *propositions; e.g.* that the middle *term* cannot be taken twice particularly is a rule of the former kind, and that from two negative *premisses* (or *propositions*) nothing follows is one of the latter. Faults of this latter kind necessarily affect modes in all figures alike, as for instance EEA is invalid in any figure for having two negative premisses; and consequently, in selecting our eleven valid modes in the last chapter, we have been able to exclude faults of this kind completely. But it is otherwise with faults of the former kind. Since the position of the terms is different in each figure, modes designated by the same vowels may be faulty in one figure and valid in another. Thus the mode AAA is invalid in the second figure, because the middle term is taken twice particularly (or it has what is called an undistributed middle); while in the other three figures

this fault is avoided. But in the third and fourth figures it has another fault, namely, that the minor term is particular in the premiss (being the predicate of an affirmative proposition), while it is universal in the conclusion—a fault which we described as illicit process of the minor term. Hence this mode is valid in the first figure only. If then we wish to ascertain what modes are valid *in each figure*, we must turn to the general rules and see how many of them affect *terms*, not propositions, and then try how many of our ten (or rather eleven) valid modes comply with these rules in each figure.

The rules which affect terms, not propositions, are (1) *the middle must not be particular in both premisses*, and (2) *no term can be particular in the premiss and universal in the conclusion*. The fault occasioned by violating the first of these rules is called *undistributed middle*, and that occasioned by violating the second is called *illicit process* of the *major*, or of the *minor* term, according as the term with which it occurs is the major or the minor. The three faults, therefore, which may affect a mode in one figure, while not affecting the same mode in a different figure, are (1) undistributed middle, (2) illicit process of the major term, and (3) illicit process of the minor term. The **special rules of the syllogism**, as they have been called (*i.e.* rules applicable to syllogisms in particular figures only), are simply rules for avoiding these faults, and they will necessarily be different in each figure, owing to the different position of the terms.

1. Take the first figure. An illicit process of the minor term cannot occur in any of our eleven selected

modes, for here (and in the second figure) the minor term is the subject of the minor premiss as well as of the conclusion, and an illicit process of the minor would require a particular minor premiss and an universal conclusion—a fault which has been excluded by the rules relating to propositions. But we require a special rule to guard against an illicit process of the major term. The major term in this figure (as also in the third) is the predicate of both major premiss and conclusion. There will be an illicit process if it is particular in the former and universal in the latter, *i.e.* if the major premiss be affirmative and the conclusion negative. But to have a negative conclusion from an affirmative major premiss, it appears by the rules relating to propositions that the minor premiss must be negative; while again, whenever the minor is negative, the major is affirmative and the conclusion negative. Hence the **first special rule**, the object of which is to avoid an illicit process of the major, in the **first figure** (and also in the third) is—**the minor premiss must not be negative**, or **the minor premiss must be affirmative.** We still require another special rule to exclude an undistributed middle. The middle term in the first figure is the predicate of the minor premiss, and that premiss, as we have seen, is affirmative. Hence the middle term must be universal in the major premiss to avoid an undistributed middle; and since (in the **first figure**) it is the subject of the major premiss, the **second special rule** emerges, viz., **the major premiss must be universal.** If then we want a list of the modes which are valid in the first figure, we go back to our original eleven and select

those only which have an affirmative minor and an universal major, and we thus obtain AAA, AAI, AII, EAE, EAO, EIO. All these are valid; but two, AAI and EAO, are evidently useless, since we can draw the conclusions A and E from the same premisses, instead of I and O.

2. Coming to the second figure, an illicit process of the minor term, as has been remarked, is already excluded. But there will be an undistributed middle unless one premiss be negative, and (since the negative premiss necessarily involves a negative conclusion) there will be an illicit process of the major term unless the major premiss be universal. Hence the **two special rules of the second figure are one premiss (and the conclusion) must be negative,** and **the major premiss must be universal.** Six again of the selected modes answer these tests, viz.—EAE, EAO, AEE, AEO, EIO, OAO—all of which are legitimate; but EAO and AEO are useless, inasmuch as in both instances the conclusion E can be drawn from the same premisses.

3. Take the third figure. We have already seen that to avoid an illicit process of the major term **there must be an affirmative minor premiss.** But in this figure the minor term is the predicate of this affirmative minor premiss, and therefore particular. Hence, to escape an illicit process of the minor term, that term must be particular in the conclusion also, *i.e.* **the conclusion must be particular.** But here none of the selected eleven can have an undistributed middle; for the middle term, being the subject of both premisses, cannot be undistributed unless

both premisses are particular, which fault has been excluded by the rules relating to propositions. Applying our two rules, we again find six valid modes—AAI, EAO, AII, IAI, EIO, and OAO. None of these are useless, because no premisses can lead to an universal conclusion in this figure.

4. Lastly, the fourth figure admits of all three faults within our selected modes, and, therefore, we require three special rules (one directed against each) to avoid them. Since the middle term is here the predicate of the major premiss and the subject of the minor, there will be an undistributed middle if the major premiss be affirmative *and* the minor premiss particular; and hence emerges our **first special rule**, which may be stated in either of the following ways:—**If the major premiss is affirmative, the minor premiss is universal**; or, *If the minor premiss is particular, the major premiss is negative.* Again the major term is here the subject of the major premiss. Hence there will be an illicit process of that term if the major premiss is particular *and* the conclusion negative; and so we reach the **second special rule**, which may be expressed either as **in negative modes the major premiss is universal**; or, *If the major premiss be particular, the mode is affirmative.* Finally, the minor term being the predicate of the minor premiss, there will be an illicit process of that term, if the minor premiss is affirmative *and* the conclusion universal. Hence the **third special rule—If the minor premiss be affirmative, the conclusion is particular**; or, *If the conclusion be universal, the minor premiss is negative.*

Applying these three restrictions to the selected modes, we find the following six are left:—AAI, AEE, AEO, IAI, EAO, EIO; but here AEO is useless, since AEE is legitimate in this figure; and as the same thing occurred in the second figure, which was the only other one in which AEO proved to be legitimate, we may lay this mode aside as being in all cases either illegitimate or useless.

Logicians have given special names to the valid modes of each figure which, preserving the vowels as before, introduce consonants which distinguish the one from the other. Thus the four useful modes of the first figure are called Barbara (perhaps the meaning would be clearer if it was written bArbArA), Celarent, Darii, and Ferio (the two useless ones, AAI and EAO, have got no such special names). The four useful modes of the second figure are called Cesare, Camestres, Festino and Baroko; the six legitimate modes of the third figure are called Darapti, Felapton, Disamis, Datisi, Bokardo, and Ferison, and the five useful modes of the fourth figure are called Bramantip, Camenes, Dimaris, Fesapo, and Fresison. These names have meanings which it may be as well briefly to explain. Aristotle, the founder of Logic, regarded a valid syllogism in the first figure as possessing a self-evidence which the valid syllogisms in the other figures did not possess. In fact, a syllogism in the first figure, with an *universal* major premiss (relating to a *whole* class) and an *affirmative* minor premiss (*including* something in that class) is but a special example of the *Aristotelian* **Dictum**, which

states the self-evident truth already stated in other terms, that—

Whatever may be affirmed or denied of a whole class,
In which class something else is included,
May be affirmed or denied of that something else

(the three parts of the Dictum, as above stated, corresponding to the major premiss, minor premiss, and conclusion respectively); * but valid syllogisms in the other figures do not fall so readily under any similar principle. Aristotle proposed to prove their validity as follows:—If I take a syllogism in the second figure (suppose), and show that *from the same premisses*—employing no process whose validity is open to any doubt—I *can deduce the same conclusion by a syllogism in the first figure*, I prove this syllogism in the second figure to have been valid. For this purpose the processes employed (if we omit the reductions of Baroko and Bokardo, which were of a roundabout and troublesome character) were two in

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* The *class* is here the *middle* term; the *major* term is affirmed or denied of *it* in the major premiss, and the minor term is stated to be included in *it* in the *minor* premiss. In affirming or denying anything of a class, we make the class the *subject*. Hence the middle term is the subject of the major premiss, which must be *universal*, inasmuch as something is to be affirmed or denied of the *whole* class. But in stating that anything is included in (or excluded from) a class, we make the class the *predicate*. Hence the middle term is the predicate of the minor premiss, which must be *affirmative*, inasmuch as the minor term is to be *included in*, not *excluded from*, the class. Thus the Dictum, as above stated, is only applicable to a Syllogism in the first figure with an universal major premiss, and an affirmative minor premiss.

number, viz., *conversion*, whose validity has been already established, and *transposition of the premisses*, i.e. turning the major premiss into a minor, and the minor premiss into a major. The validity of the last process is evident, for if the premisses are true when stated in one order, they must be equally true when stated in the other. If, however, the premisses are transposed, the new conclusion will not be identical with the old one, for the minor term of the transposed syllogism (*i.e.* the former major) will be its subject, and the former minor its predicate. But, says Aristotle, it may be *converted* into the old conclusion, and thus the validity of the old syllogism is proved, the *old conclusion* being *ultimately* deduced from the *old premisses.* Hence conversion and transposition are the only processes employed; but we may have to convert the *new* conclusion as well as one or both of the *old* premisses. Now each of the names we have given includes three of the vowels A, E, I and O, the first of which stands for the Major Premiss, the second for the Minor Premiss, and the third for the Conclusion. The names of the modes of the first figure are, with the exception of these vowels, insignificant; but it is otherwise with those of the modes of the second, third, and fourth. These names indicate to what mode of the first figure they can be reduced, and how the reduction is to be effected. The first letter in the names of these modes indicates that they are reducible to the mode of the first figure the name of which begins with the same letter, *e.g.* Bramantip to Barbara, Darapti to Darii, Cesare to Celarent, Festino to Ferio. *S* stands for

simple conversion, or conversion in which the quantity and quality of the proposition remain unchanged, and *p* for conversion *per accidens*, or conversion in which the quality is preserved but the quantity is diminished; and these letters are placed immediately *after* the vowel standing for the premiss which is to be converted; but if the new conclusion is to be converted, then *s* or *p* forms the last letter of the name. Finally, *m* indicates that the premisses are to be transposed for the purposes of reduction. The reader will, perhaps, find some amusement in working out these reductions, but on the principles we have laid down they become unnecessary.* Other logicians, moreover, have laid down Dicta as self-evident (or almost as self-evident) as that of Aristotle, which apply directly to the valid modes of the second, third, and fourth figures, and thus supersede the necessity of any reduction.

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* The name Bramantip, designating the mode AAI of the fourth figure, will probably suggest to the reader the question, How can the conclusion I be converted *per accidens*? Recollecting, however (as the letter *m* reminds us), that the premisses have been transposed, the letter *p* indicates that it is the *new* conclusion which is to be converted *per accidens* in order to arrive at the *old* conclusion I. The new conclusion, therefore, is not I, but a proposition which can be converted *per accidens* into I; *i.e.* it is A, as we might have inferred from the initial letter B.

CHAPTER VIII.

SORITES, AND HYPOTHETICAL AND DISJUNCTIVE SYLLOGISMS.

We have now a complete scheme or table of all valid arguments in which a conclusion is drawn from two categorical premisses; while longer reasonings can be reduced to a series of syllogisms, and their legitimacy or illegitimacy ultimately tested by the same table. Thus, if we had an argument of the following form:—

All B is C;
All C is D;
All D is E;
All E is F;
∴ All B is F;

we can make of it these three Syllogisms—

1	2	3
All C is D.	All D is E.	All E is F.
All B is C.	All B is D.	All B is E.
All B is D.	All B is E.	All B is F.

Such an argument is called a **Sorites**, and the one before us is a valid argument, since we can derive the same conclusion from the same premisses by three syllogisms in Barbara. Logicians have, however, dis-

covered means of testing the validity of a Sorites directly, and thus dispensing with its reduction to a series of syllogisms. In the Sorites it will be observed that the subject of each proposition (except the conclusion or last proposition) is the predicate of the preceding one, and that the conclusion (when affirmative) affirms the predicate of the second last of the subject of the first. The rules for an argument so constructed **are, that no proposition previous to the second last can be negative,** and **no proposition intermediate between the first and last can be particular.*** These rules correspond to those of the first figure, viz., that the minor premiss cannot be negative nor the major premiss particular; for it will be observed that the *second* proposition of the Sorites is the major premiss of the first syllogism, the first proposition being the minor premiss of the same. The Sorites can also be brought under a Dictum corresponding to that of Aristotle, viz.:—Whatever may be affirmed or denied of a whole class may be affirmed or denied of anything that is wholly included in a third thing, which third thing is wholly included in the class—adding, of course, a fourth, fifth, or sixth thing, if the number of propositions in the Sorites be five, six, or seven, &c. The stress of the argument lies on the total inclusion of each of

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* There is another kind of Sorites sometimes known as the Goclenian Sorites, in which the predicate of each proposition is the subject of the preceding one. The reader will easy see how these rules should be modified in order to apply to it.

the terms in the preceding one, and hence all the propositions except the first and last will be universal affirmatives (for a negative proposition asserts an *exclusion*, not an inclusion).

But there are other kinds of reasoning which seem to escape our rules altogether, namely, those which proceed from hypothetical or disjunctive premisses. For instance,

> If the barometer falls, it will rain;
> The barometer is falling;
> Therefore it will rain.

But if we recollect that a term in Logic may include many words, we shall see that such reasonings can be brought under the principles already laid down. The major premiss is here equivalent to All cases-in-which-the-barometer-falls are cases-in-which-it-is-about-to-rain. The minor premiss may be written, The present-case is a case-in-which-the-barometer-falls, and the conclusion will be, The present-case is a case-in-which-it-is-about-to-rain.* This is a syllogism in Barbara; for the singular proposition, The present-case is a

* It is always understood that a hypothetical proposition asserts some relation either of cause and effect or of reason and consequent between its parts, which almost disappears in the categorical equivalent. Still, if the latter be true, it can hardly be contended that the former is not so. I have therefore not deemed it necessary to examine further the nature of what has been termed the *vis consequentiæ* in a hypothetical proposition. The equivalence, however, is more perfect when the hypothetical proposition is general than when it is singular.

case-in-which-the-barometer-falls, may be treated as an universal. But if we once ascertain under what conditions hypothetical reasonings are valid, we may dispense with this awkward reduction in individual instances; and this is easily done. From the hypothetical proposition, *If B is C then D is F*, there appear to be only four possible ways of arguing, viz:—

I.	II.	III.	IV.
If B is C, D is F; B is C; ∴ D is F.	If B is C, D is F; D is F; ∴ B is C.	If B is C, D is F; B is not C; ∴ D is not F.	If B is C, D is F; D is not F; ∴ B is not C.

Turning these into categorical syllogisms in the way already indicated, we obtain—

I.

Every case of B being C is a case of D being F;
Every possible case is a case of B being C;
∴ Every possible case is a case of D being F.

This is a good syllogism in Barbara, the terms being, *major*, case-of-D-being-F, *minor*, possible-case, *middle*, case-of-B-being-C. Hence in hypothetical syllogisms **it is legitimate to argue from the affirmation of the antecedent to the affirmation of the consequent.** (It is hardly necessary to say that in the proposition, *If B is C, D is F*, the antecedent is *B is C* and the consequent *D is F*.)

II.

Every case of B being C is a case of D being F;
Every possible case is a case of D being F;
∴ Every possible case is a case of B being C.

This syllogism is in the second figure, and is invalid for undistributed middle, the middle term being case-of-D-being-F. Hence in hypothetical syllogisms **it is not legitimate to argue from the affirmation of the consequent to the affirmation of the antecedent.**

III.

Every case of B being C is a case of D being F;
No possible case is a case of B being C;
∴ No possible case is a case of D being F.

This is a syllogism in the first figure with a negative minor, and therefore invalid for an illicit process of the major term, viz.: case-of-D-being-F. Hence **it is not legitimate to argue from the denial of the antecedent to the denial of the consequent.**

IV.

Every case of B being C is a case of D being F;
No possible case is a case of D being F;
∴ No possible case is a case of B being C.

This is a good syllogism in Camestres of the second figure, the middle term being case-of-D-being-F. Hence **it is legitimate to argue from the denial of the consequent to the denial of the antecedent.**

These are the **four laws of hypothetical reasoning.** It will be seen that a hypothetical proposition is always the equivalent of an universal affirmative, which simplifies its rules considerably. Occasionally, however, the hypothesis is carried on further; *e. g.*,

If B is C, D is F;
If D is F, G is H;
∴ If B is C, G is H.

This is a valid syllogism in Barbara according to the above system of reduction, and can be brought under one of the foregoing rules by a slight amendment, viz.:—*It is legitimate to argue from the (hypothetical). affirmation of the antecedent to the affirmation of the consequent (on the same hypothesis).* This takes in the above syllogism, considering *If D is F, G is H,* as the proposition reasoned upon. *Its* antecedent (D is F) is affirmed on the hypothesis that B is C; therefore, *its* consequent (G is H) may be affirmed on the same hypothesis.

Another kind of proposition apparently excluded from our rules is that known as disjunctive, such as—Virtue either tends to procure the approval of mankind or the favour of God. But propositions of this kind can be very easily put into the hypothetical shape, and the rules last arrived at applied to them. The proposition given above is the precise equivalent of either of the following:—If virtue does not tend to procure the approval of mankind, it tends to procure the favour

of God; or if virtue does not tend to procure the favour of God, it tends to procure the favour of mankind.* Logicians, however, generally admit the validity of the following inference:—

Either B is C or D is F;
D is F;
∴ B is not C.

If we put this into the hypothetical form and then reduce it we obtain

Every case of B not being C is a case of D being F;
Every possible case is a case of D being F;
∴ Every possible case is a case of B not being C;

which is plainly invalid for undistributed middle. But it has been supposed that a proposition of the form—*Either B is C or D is F*, implies that the two propositions B is C and D is F cannot *both* be *true*, and if so, the above disjunctive reasoning would be correct. But though the speaker may often intend to imply by the words "either"—"or" that both alternatives cannot be simultaneously true, these words do not necessarily imply it, and therefore it seems to me that this mode of reasoning is invalid. In the example above, for instance, it would be at least possible that virtue tended to pro-

* These hypotheticals are not two distinct propositions, being, in fact, the converses (by contraposition) of each other. They convey exactly the same meaning.

cure *both* the approval of mankind and the favour of God. However, even assuming the contrary, our rules of reduction do not cease to apply. If the words "either"—"or" necessarily imply mutual exclusiveness, the disjunctive proposition is equivalent not to one but to two hypotheticals—viz: If B is not C, D is F, and If B is C, D is not F; and it is the second of these hypotheticals that forms the major premiss of the reasoning in question—viz.:

Every case of B being C is a case of D not being F;
No possible case is a case of D not being F;
∴ No possible case is a case of B being C;

which is a good syllogism in Camestres; but we have assumed that its major premiss is really implied by the disjunctive proposition, Either B is C or D is F. There are, of course, other varieties of hypotheticals and disjunctives, such as, for example—If B is C, either D is F or G is H, but they do not require special treatment. The rules of Disjunctive reasoning can be easily inferred from what has been said. They are—**From the denial of one or more members of the disjunctive premiss we can infer the truth of the remaining one** (*or of some one of the remainder where more than one are left*): and if the second mode of inference is deemed valid, *From the affirmation of one member we can infer the falsity of the other or others;* and of course a similar result follows from the alternative affirmation of more than one. This latter rule, however, I do not admit.

Another variety of the hypothetical syllogism is what is known as an **Enthymeme**,* in which the conclusion is apparently drawn from a single proposition, the additional premiss (which is not expressed in terms) being, *If the (expressed) premiss is true, the conclusion follows.* Thus, if I argue, This country is distressed, therefore it is under a tyranny, the full reasoning plainly is: If this country is distressed, it is under a tyranny; but it is distressed; therefore it is under a tyranny. We can often simplify such reasonings by supplying a categorical premiss instead of a hypothetical. Thus, supposing the argument to be, Every animal has a nervous system, therefore Every man has a nervous system, it is pretty plain that Every man is an animal, is the link to be supplied. This cannot always be done, however, and one of the commonest modes of misrepresenting an enthymematic argument is to fill up the links categorically where such was not the reasoner's intention. For instance, one of the arguments employed against the great antiquity of the Homeric poems is that no early imitations of them exist. I have seen this enthymematic reasoning filled up by an opponent thus: Every poet imitates the writings of every preceding poet—as if that was the only supposition on which Homer would have found imitators. It may be remarked that in an argumentative treatise the author very rarely expresses *all* his premisses; and to judge accurately of the reasoning you

* Aristotle uses the word Enthymeme differently, but his distinction is useless in Logic.

should be very careful as to the wording of the premisses which have to be supplied. What a writer *says* is often misrepresented, but it is still more common to misrepresent what he does not say, but implies.

Other ways of expressing propositions will readily occur to the reader. For example, a question is very often asked where the real premiss on which the following reasoning depends is the supposed answer to it. In dealing with such an argument logically we must substitute this supposed answer for the question. So, an exclamation or sentence terminating with a note of admiration often forms a premiss in reasoning, and we must here also supply its logical equivalent, if we wish to test the reasoning. Sometimes, too, an adjective or epithet in a sentence can be developed into a proposition, and one on which the train of reasoning depends. When recourse is had to ridicule, the real premiss (or rather reasoning) is, This doctrine is ridiculous, and therefore untrue.* Logic can lay down no general rules for reducing the ordinary modes of expression to logical forms. The reader must do that for himself; and it is only when he has done it that Logic can tell him whether the reasoning is good or bad. But for this purpose it is by no means necessary to reduce the reasonings in question to a series of syllogisms. It is never necessary to go beyond the reduction to a Sorites, the rules for which have been given in this article; and it is frequently not necessary to go so far.

* But it does not follow that everything is ridiculous which may be exhibited in a ludicrous light—possibly by means of some false analogy.

We have now obtained an exhaustive enumeration of all the species of good reasoning, and I think it may be safely laid down that any argument that cannot be reduced to one or other of them is invalid. Take care, however, before saying that an argument cannot be reduced, to ascertain whether it is expressed *in full:* for enthymemes are very common, and, when the unexpressed premiss is some very obvious truism, we are apt to forget that anything is left unexpressed. For example, an argument of this form—

Three-fourths of the army were killed;
Three-fourths of the army were Prussians;
∴ Some of the Prussians were killed;

is logically incomplete, and to complete it we must state in terms that any two fractions of the same whole, each amounting to three-fourths (or to over one-half) must have a common part—a truth so obvious that few persons would think it necessary to state it, but which, nevertheless, is taught by arithmetic, not by logic. This would become evident if we introduced fractions of a more complicated character, when the reader would probably find it necessary to work out a sum in arithmetic before he could determine whether the conclusion followed from the premisses or not. On supplying the arithmetical premiss or premisses the argument becomes logically valid, and the appearance of an undistributed middle is removed. In fact, all *completed* reasonings, whether good or bad, can be thrown into logical form—the only difference being that

good reasoning will fall into some of the valid modes, and bad reasoning into some of the invalid ones. But we may reason rightly from false premisses, or wrongly from true ones; and the conclusion is not proved unless the reasoning is valid *and* the premisses true. If the first requirement is fulfilled, it is usually desirable to look to the second.*

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* This is perhaps the best place to explain the logical character of a *Dilemma*. It is a Syllogism in which the Major Premiss is a hypothetical proposition with a disjunctive consequent, both branches of which consequent are denied by the minor. Thus—

If B is C, either D is E or F is G;
But D is not E, and F is not G;
∴ B is not C.

The name Dilemma, however, is sometimes extended to a Syllogism of the following form—

If B is C, either D is E or F is G;
If D is E, H is I, and if F is G, K is L;
∴ If B is C, either H is I or K is L.

When an argument of this kind is called a Dilemma it is supposed to be used against some one who admits that B is C, but is unwilling to admit either that H is I or that K is L. He is thus placed on "the horns of a dilemma," *i.e.* he must select which admission he will make. Sometimes it is simplified thus—

Either D is E or F is G;
If D is E, H is I, and if F is G, K is L;
∴ Either H is I or K is L.

Any of these forms will be found to fall within the preceding rules.

CHAPTER IX.

FALLACIES.

It is no easy task to treat of Fallacies within a moderate compass, especially as writers on Logic are not agreed as to what is to be called **a Fallacy**. The distinction which I would draw is a very simple one. I would define a fallacy **an invalid argument,** or an argument in which the conclusion does not follow from the premisses. (Perhaps I should have written "premiss or premisses," for I see no reason why such reasonings as the following should not be classed as fallacies—All men are mortals, therefore, All mortals are men—If the country is under a tyranny it must be distressed; therefore, If the country is distressed it must be under a tyranny.) Most logicians, however, employ the term fallacy in a wider signification.

Now, in treating of valid arguments, it might appear that we had sufficiently dealt with invalid ones also. If an universal affirmative proposition (A) can only be converted into a particular affirmative (I), then its conversion into an universal affirmative is a fallacy. If a syllogism, containing an undistributed middle, or an illicit process of either the major or the minor term (or, of course, a syllogism manifestly containing four terms) is inconclusive, it is a fallacy; and the rules for the reduction of hypothetical and disjunctive syllogisms to the categorical form will show when these latter syllo-

gisms are fallacious. And, in fact, the largest class of fallacies is excluded by the rules already laid down, namely, what Archbishop Whately calls Logical Fallacies. Of these it is only necessary to say here, that sometimes an argument in which there are three propositions, with apparently four or five terms, will, by a mere change in the form of expression, be reducible to one in which there are only three terms, and which, on examination, will prove perfectly valid; while, on the other hand, syllogisms, which apparently have three terms only, will often turn out to have really four, and will consequently fall into the class of fallacies. For, in the first place, the same proposition (or the same idea) can often be expressed in very different terms, whereas in judging of the validity of an argument we must attend to the meaning intended to be conveyed, and not merely to the words conveying it. Thus in the following argument:—

No irrational agent could have produced a work which manifests design;

The Universe is a work which manifests design;

∴ The Universe is the work of a rational agent,

the reasoning is valid, for the major premiss is the equivalent of

Every work which manifests design is the work of a rational agent;

though, from the different mode of expression, the reader might, at first sight, think there were four or

five terms. On the other hand, in the following argument:—

Every one desires happiness;
Virtue is happiness;
∴ Every one desires virtue,

we can very easily reduce the terms (apparently) to three, by reading the major premiss "(All) happiness is a thing desired by every one," with a similar change in the terms of the conclusion. But on a closer examination of the minor premiss, it becomes clear that the speaker does not literally mean that virtue *is* happiness. He intends to imply only that happiness is the *effect* or the *invariable accompaniment* of virtue; and, as soon as what is meant is properly expressed, we find that there are four terms in the argument which do not admit of any further reduction. For it is evident that I may desire the effects or accompaniments of a thing without desiring the thing itself, and *vice versâ*. The reader should be on his guard against cases of this kind. Nothing is more common with orators, or with inaccurate reasoners, than to employ the word *is*, not in its proper logical signification of identity, but to express the relation of cause and effect, or that of co-existence. So, too, the present tense is sometimes used when the past or future should have been employed in order to convey accurately the intended meaning, as in the syllogism—

He who is most hungry eats the most;
He who eats least is most hungry;
∴ He who eats least eats most.

Here what is meant by the major premiss is, He who is most hungry *will eat* most, while the minor means, He who *has eaten* least is most hungry, which two premisses lead to the legitimate conclusion that he who *has eaten* least *will eat* most. In fact, the proportion of current fallacies, which arise from the admission of inaccurate and elliptical modes of expression into arguments, is a very large one. Take the following as another example:—

> Two and three are even and odd;
> Two and three are five;
> ∴ Five is even and odd.

Here the major premiss is false. Two is even and Three is odd, but in no admissible sense is it true that Two and three are even and odd. The reader, however, readily accepts the major premiss, because he regards it as an abbreviated statement of *two* propositions both of which are true, and he never dreams of questioning it until he finds it reasoned upon as a *single* proposition. It is, therefore, a primary rule in judging of the soundness or unsoundness of any argument, to see, in the first place, that what is meant to be stated is expressed with all the fulness and accuracy that language admits of. No figure of speech, however natural or universal, can be admitted into an argument which is to be tested by the rules of syllogism.*

* Another (logically) inaccurate mode of expression in common use is exemplified by the proverb, All that glitters is not gold. Here the real

But the rules of syllogism do not sufficiently protect us against one extensive class of bad reasonings—those founded on ambiguities in the terms employed, and especially of the middle term. Owing to this, an argument may appear at first sight perfectly conformable to the rules of syllogism, when it is really not so. This ambiguity may sometimes affect the entire proposition, but it more commonly affects one of the terms only, in which case (so soon as the ambiguity is detected) the fallacy will fall under the head of a syllogism with four irreducible terms. Ambiguities are of various kinds. Sometimes the same word stands for two or more distinct ideas in its ordinary use, as happens, for example, in the case of Light or Post. Reasonings based on ambiguities, however, can deceive nobody, unless there is some connexion between the two meanings, which may lead to a confusion between them; as, for example, occurs with the term Law, the scientific meaning of which is not very different from its ordinary signification, while the senses in which it includes and excludes what is called Equity are seldom distinguished except by jurists. But where the words are not in themselves ambiguous, they may be differently employed in the two premisses, and thus become ambiguous from the context. The most common instance of this is

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meaning is not *All is not*, but *Not all is*, which is a particular, not an universal, proposition. Indeed it seems meant to include *two* particular propositions, viz.:—Gold glitters (something that glitters is gold), and, Other things than gold glitter also (something that glitters is not gold).

where the middle term is used collectively in one premiss, and distributively in the other, *e.g.*,

> part
> Things which happen every day are not improbable;
> part Some things, against which the chances are millions to one, happen every day;
> ∴ Some things, against which the chances are millions to one, are not improbable.

Here the term "things-which-happen-every-day" is used distributively in one premiss, and collectively in the other, which, indeed, also occurred in our former case of Two and three being even and odd. I think it will be found on investigation, however, that in all cases where the ambiguity is not in the middle term itself, a more accurate statement of the premisses will bring the fallacy to light. Here the premisses correctly expressed are:—

> If the *same* thing happens every day, it is not improbable;
> On every day something happens, against which the chances are millions to one.

from which premisses no conclusion follows; for while something against which the chances were millions to one happens every day, it will be a *different* thing on each day. So again (to take another kind of ambiguity arising from the context),

> What is sold in the market is eaten; Every thing
> Raw meat is sold in the market; one thing.
> ∴ Raw meat is eaten,

the conclusion is perfectly correct;* but from carelessness of expression we are apt to take it as meaning not that Raw meat is eaten, but that Meat is eaten raw. The latter conclusion would not follow, unless the major premiss had asserted that What is sold in the market is eaten *in the same condition in which it is sold.* Had the argument been the following:—

What is sold in the market is eaten;
Wheat is sold in the market;
∴ Wheat is eaten,

no one, I presume, would have quarrelled with the conclusion, though the wheat has to be ground into flour, and baked before it is eaten. Accuracy of expression will, therefore, reduce all fallacies (in syllogisms) to a violation of some one or more of the rules of syllogism† previously laid down (including the rule

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* *i.e.* if *What* is sold in the market means *Everything* that is so sold, which, generally speaking, is not the case.

† A syllogistic term, of course, need not consist of a single word, and the ambiguity may be in the entire phrase, and not any particular part of it. Thus:—

Meat and drink are necessaries of life;
The revenues of Vitellius were spent on meat and drink;
∴ The revenues of Vitellius were spent on necessaries of life.

Here the truth appears to be, that (taking meat to mean solid food) a certain amount of meat and drink of some kind is necessary to life. No kind of meat and drink is absolutely necessary, while any kind becomes necessary *if there is no other to be had.* So far they all stand on the

against four irreducible terms), except when the middle term is in itself ambiguous; and, even in this latter case, the same reduction can be made as soon as the ambiguity is detected, while Logic can lay down no infallible rules for detecting ambiguities.*

Before quitting this subject, I ought, perhaps, to refer to what Archbishop Whately calls the fallacy of Paronymous Words. It arises entirely from the prevailing habit of stating arguments in a form different from the syllogistic. I have already noticed that in such cases the reasoning may be perfectly valid, though the syllogism, as expressed, contains four or five terms, because they are (with a little trouble) reducible to three. But there are also numerous instances in which such reasonings are invalid; then invalidity, arising not only from the employment of inaccurate expressions already mentioned, but also from the fact that words with a common

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same footing. But the phrase "necessaries of life" has got another meaning in which "necessaries" is opposed to "luxuries," and "the necessaries of life" means a sufficient amount of the *cheaper kinds* of meat and drink to support life. In the argument before us, the phrase "necessaries of life" is evidently intended to be used in one sense in the premiss and another in the conclusion; in which case the syllogism has four irreducible terms. If, however, we use it in the conclusion in the same sense as in the premiss, we have either a false premiss or a true conclusion—the former, if the necessity spoken of is intended to be absolute—the latter, if conditioned in the manner already explained.

* A practical rule which is often useful in detecting ambiguities is to translate the propositions you are dealing with into any other language you are acquainted with, and try whether three terms will still serve the purpose.

etymology sometimes have quite distinct significations. Thus—

> To have been on unfriendly terms with a murdered man is a *presumption* of guilt;
> A B was on unfriendly terms with the murdered man;
> ∴ We may *presume* he is guilty.

Here "presumption" is used in a quite different sense from "presume," and, therefore, the conclusion does not follow. Before we "presume" that a man is guilty of murder, we must not only have some "presumption" of his guilt, but one which is sufficient to outweigh all presumptions on the other side. Such reasonings, therefore, substantially come under the head of ambiguous middle.

The result of our investigations, then, has been to add *ambiguous* terms, or rather ambiguous *middle*, to the classes of fallacies previously discussed in treating of valid arguments; for, of course, when an entire proposition is ambiguous, the middle term must be affected by the general ambiguity. In fact, it is then doubtful what the middle term is. We have further seen that the division of fallacies of ambiguous middle into those where the ambiguity is in the term itself, and those where it arises from the context, is unnecessary, because in the latter case a more accurate mode of expression will always remove the ambiguity. Logicians, however, have so generally included two other species of arguments among fallacies, that it becomes necessary to

notice them. These are the (so-called) fallacies of *Petitio Principii* and *Ignoratio Elenchi*, which have not hitherto been touched on.

CHAPTER X.

PETITIO PRINCIPII AND IGNORATIO ELENCHI.

According to the view which I have taken of Logic, it treats of *conclusive* inference only. Either the conclusion is proved, or it is not proved. There is no such thing as premisses proving the conclusion to be probable. Of course if the premisses themselves are only probable, the conclusion will not be certain, and the conclusion may be false if the premisses (or either of them) are so; but so long as the conclusion follows certainly *from the premisses*, the *argument* is valid. We have proved that *if* the premisses are true the conclusion is true, and that is all that can be effected by reasoning, in the sense in which I have hitherto been using the term.

That the premisses involve or imply the conclusion is therefore a property of all conclusive reasonings; and if we were to class all arguments which possess this property as fallacies, the distinction between valid and invalid reasoning would be entirely lost. It may not convey any new truth to infer from *All men are mortal* that *Some mortals are men*, or to infer from *All men are mortal*, and *The Duke of Wellington is a man*,

that *The Duke of Wellington is mortal;* but in no admissible sense of the word is either of these reasonings a *fallacy*. There is, however, this difference between an immediate inference and a syllogism, that the former never can lead us to any new truth, while the latter may. The reason of this difference is, that in the former case we reason from a single proposition, and the inference drawn from it (by conversion, subalternation, obversion, or opposition) is so clearly implied in its very meaning, that it is impossible to suppose anyone who thoroughly understands the one not to admit the other. But to draw a syllogistic conclusion, it is not only necessary to have a general (or habitual) knowledge of both premisses, but also to put them together; and a great many people never put together the different portions of their knowledge, in order to see what consequences they lead to. For example, we may suppose a man to know the ordinary tests for arsenic, so as to be able to answer the question if asked at an examination, and yet not to think of arsenic, or the tests for its presence, when they are visibly fulfilled before him, and so to miss the discovery that the substance he is looking at contains arsenic. The principal obstacle to following such a book as Euclid is the difficulty of bringing together different portions of our previous knowledge rapidly and clearly; and many writers describe all the processes of Pure Mathematics as consisting only in the proper selection and putting together of the portions of our previous knowledge which are best calculated to lead to results that have not hitherto been thought of. This

doctrine is indeed untenable in its full extent. No combination of our previous knowledge would give us the *constructions* which in many cases are necessary to the proof of Euclid's theorems; but where the proof does not require any lines to be drawn except those mentioned in the enunciation of the theorem, the whole difficulty, either in making the original discovery, or in following the proofs,* arises from the fact that all our knowledge can never be simultaneously present to the mind, while the discovery or proof requires certain specific portions of it to have been simultaneously present and thought of in conjunction.

I have made these remarks as a preliminary to considering the so-called fallacy of *Petitio Principii*, or assuming that which ought to be proved. In my view there is no such fallacy. What is sought to be proved *must* be assumed in the premisses, or else, strictly speaking, it is not proved at all; for reasoning or proof (as I have hitherto considered it) simply consists in showing that the premisses necessarily imply the conclusion. But though there is no such *fallacy* as a *Petitio Principii*, there is an objectionable form of argument to which that name may be properly applied. This objectionable form consists in endeavouring to palm off as a syllogism what is really an immediate inference, and thus to induce the reader to believe that he is being led to some new truth, when, in reality, a previous allegation

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* Supposing, of course, that the discoverer or learner is previously aware of all the propositions assumed in the proof.

or assumption of the reasoner is merely disguised by expressing it in different terms. It would not be a *fallacy* to argue—

All men are mortal;
Some men are men;
∴ Some men are mortal;

but the reader would consider it absurd to attempt to prove by a syllogism what was evident to everyone who understood the major premiss, and he would regard the minor premiss as wholly superfluous. Now, if we bear in mind the distinction formerly drawn between analytical and synthetical judgments—or propositions in which the predicate forms a part (or the whole) of the meaning of the subject, and propositions in which the meaning of the predicate is distinct from that of the subject—it will be at once seen that no number of analytical propositions can be of any use in arriving at a new truth. They, in fact, merely explain (completely or partially) some of the terms previously used in the argument; and to one who fully understands the meaning of those terms they are of no use whatever. Hence, if a syllogism has one analytical premiss, the conclusion can never contain any assertion which is not included in the *other* premiss, and the argument therefore is not really a syllogism, but an immediate inference, usually disguised by employing language different from that used in ordinary subalternation, conversion, and opposition. This kind of argument, though of little use, is not in itself fallacious; but it is often used unfairly,

the reasoner endeavouring to conceal his assumption of the synthetical premiss, by assiduously directing the attention of the reader or hearer to the analytical one. It is, therefore, a good rule, in judging of arguments, to be more than ordinarily suspicious when the reasoner devotes all his attention to enforcing some self-evident or very-well-known truth, or assumes that anyone who differs from him must deny it. **A Petitio Principii** I would define as **A syllogism in which one premiss is an analytical proposition.** It is not a fallacy, but it leads to no new truth; and whoever doubts the conclusion is sure (if he sees and understands it) to doubt the synthetical premiss; for the former is either a part of the latter, or else the very same proposition differently expressed.*

Ignoratio Elenchi, on the other hand, generally speaking, *is* a fallacy, namely, an ambiguous middle (or some other logical defect) in the *second* syllogism of an argument. It has two forms. In the first, I undertake to prove a certain proposition and I then prove *another* proposition sufficiently like it to be mistaken for it by the reader or hearer. The second is of a similar character. I prove one proposition and in my subsequent reasonings take it for granted that I have proved a different proposition sufficiently like it to enable the former to be mistaken for the latter. In both cases there is one valid syllogism and another syllogism

* A Syllogism in which one premiss is some very-well-known truth possesses many of the same properties as a Syllogism one of whose premisses is an analytical proposition.

(usually implied rather than expressed) which falls within some class of fallacies previously enumerated; though the second syllogism may be valid also, if the truth of one of its premisses is open to doubt.

A considerable number of these fallacies turn on the ambiguity of the word "right" as implying either a *legal* right or *moral* rectitude. The great defence of all kinds of oppression is that Mr. So-and-so is only enforcing his *rights*, which it is contended that he cannot be *wrong* in doing. But it does not seem to be possible to draw up a Code of Laws which will not in some instances permit a person to enforce what is morally wrong; and it is certain that every existing Code can be occasionally made use of as an instrument of oppression and injustice. These fallacies would usually become more evident if the *whole* argument (including both what was proved and what was subsequently reasoned upon) was stated in the form of a Sorites. It would then be seen at once that a link was unsupplied.

The other so-called fallacies mentioned by Logicians are mere artifices for inducing the hearer or reader to accept premisses of which there is no satisfactory proof; or to accept an alleged conclusion as a consequence of certain premisses, without examining whether it really follows from them. It is very desirable to be on one's guard against such artifices, and to know a few of the most ordinary "dodges" of the kind which are exposed in Whately's Logic and similar works. Whately, moreover, exemplifies some of them in his own person. "*As if*" is with him (and others) a favourite way of

introducing a refutation of some theory by its supposed consequences, or by a supposed parallel case, when it will perhaps be found on investigation that there is no relation of consequence or parallelism between the two propositions. And this leads me to observe, that I believe there are fewer fallacies current in the world than is sometimes alleged. When we wish to resist the conclusion arrived at by any reasoner, we must either dispute one (or both) of his premisses, or else charge his reasoning with fallacy. But to dispute the premisses (though it might frequently be done with success) requires not only a knowledge of the subject, but often a good deal of patient thought; whereas a charge of fallacy can be made by anyone acquainted with the rules of Logic, though totally ignorant of the facts. The most convenient mode of refutation then is to detect a fallacy in the argument; which is done by representing its author as saying what he never intended to say, or (as an argumentative writer seldom expresses all the links in his chain of reasoning at length) filling up the gaps in his argument in a way that he never dreamt of. When Sir William Hamilton meets with an obnoxious doctrine in Philosophy, he usually contends that in "ultimate analysis" it is "self-contradictory;" and various other writers adopt the same mode of confutation. It is a good rule, whenever you find a distinguished writer charged with falling into some gross fallacy, to examine what he *has* said instead of what he is represented to have said, and in nine cases out of ten the alleged fallacy

will disappear.* He may have reasoned from false premisses and so have arrived at a wrong conclusion, but the reasoning itself is usually valid. When it is not so, the most common form of error is that which has just been described as *Ignoratio Elenchi.*

Whether included under the name of Logic or not, however, there is another branch of inquiry which is necessary to complete the problem, namely, *Probabilities.* Logic, as I have expounded it, treats of conclusions which follow certainly from the premisses, and the only way in which such a conclusion can be probable is that it follows from a premiss which is only probably true. But the reader will ask how do we come by the probable premiss or premisses of this Syllogism. And here it must be admitted that if we include under the term Proof the proof

* Except where a man has been brought up in some system of belief which he is called on in mature years to defend by argument. In this case, however good the cause may be, he frequently adopts a fallacious line of defence, or at best falls back on a *Petitio Principii.* Fallacies, too, may be fallen into in defending conclusions, the proof of which we once knew but have since forgotten; and the more familiar we are with such conclusions, and the more completely they are interwoven with our ordinary speculations and practice, the more probable it is that the train of thought by which they were originally arrived at will have escaped our memory. Hence the most important truths are often defended on fallacious grounds; and nothing can be more illogical than to reject a proposition because bad arguments are sometimes used in its favour, especially if these arguments take the shape of a *Petitio Principii.* The beliefs for which ordinary men cannot give a reason (without falling into fallacy) are often better founded than those for which they can.

that a proposition is probable, the Syllogism is not a complete theory of Proof. The certainty of a proposition may sometimes be self-evident; but its probability never can. A proposition of the form, *It is probable that B is C*, is invariably the result of evidence; and though the probable proposition which we happen to be thinking of at the moment may be the conclusion of a syllogism (with a probable premiss), it is plain that if we go back we must at last come to a probable proposition which is not the conclusion of a syllogism, but whose probability must nevertheless have been established by evidence. This, however, would lead us into a very wide field of inquiry, and one in which even the elementary principles do not seem to be fully determined; while in any event the province of Deductive Logic is essentially distinct. I do not therefore propose to enter upon any such investigation in the present Treatise.

SUPPLEMENTAL CHAPTERS.*

CHAPTER XI.

THE DEFINITION AND PROVINCE OF LOGIC.

IT is usual for the author of a treatise on any special science to commence by defining the science of which he treats; and as Definition has been one of the subjects treated of by Logicians, good definitions are naturally expected from them. Accordingly various definitions of Logic have been framed at various times by different writers. These definitions differ in two respects—as to whether Logic is a Science, an Art, or both, and as to what it is the Science or Art of. The distinction between Science and Art, when we pass beyond the productive or creative arts, is, however, by no means clearly drawn, more especially when the sciences themselves are divided into speculative and practical. Logic is clearly not a productive or creative art. It does not produce terms, propositions,† or reasonings. It merely analyzes, examines, and tests those which are produced otherwise. Further,

* These chapters are intended for more advanced students.

† Except when drawing a conclusion not heretofore enounced from previously known or accepted premisses.

the term art is usually applied to a system of rules which, in a mere treatise on the art, are left unexplained, though often borrowed from some science or sciences where their explanation can be found. Thus children learn and practise the rules of Arithmetical art without understanding the reasons for them, and a new light often flashes on the learner when he discovers this explanation in the science of Algebra. It would therefore seem improper to describe Logic as an art; and I think the reader of the preceding pages will regard the principles laid down in them (if well-founded) as strictly scientific. The distinction between a speculative and a practical science has been said to be, that in the one *scimus ut sciamus*, in the other *scimus ut operemur*. This distinction turns rather on the object of the student than the contents of the science itself. The ordinary student studies Euclid (so far as the study is voluntary) merely for the sake of the speculative information it affords; but he who intends to become an engineer or land-surveyor does so with a view to the practice of his profession. So likewise we may study Logic merely for its own sake as a science, or with a view to correct bad reasoning in ourselves, and to detect it in the productions of others. But with these ulterior objects the Logician has nothing to do. His scientific theory would be exactly the same if it were impossible to reason badly. He may insist on the utility of his science with the view of engaging the attention and interest of his reader, but the principles he is laying down would not be less true or less scien-

tific if all men reasoned correctly without their aid. Logic is, to the expounder of it, a speculative science (if the distinction between a speculative and a practical science is to be admitted), and it is from this point of view that he should define it. Some writers indeed distinguish between *Logica Docens* as a speculative science, and *Logica Utens* as a practical one. I have never met with a treatise exclusively appropriated to *Logica Utens*, and I confess that I hardly understand how such a treatise could be written. At all events the present work is not of that character. I have endeavoured to deal with Logic as a speculative science; and the remarks intended to have a practical bearing are not more numerous than what the student might expect to meet with in a scientific treatise on Heat, Sound, or Electricity.

But what is Logic the Science of? The current definitions differ widely in this respect. Before enumerating them, however, I may briefly dispose of a modification of some of these definitions which is often put forward as important. Instead of using such expressions as The Science of Reasoning, or The Science of Thought, the writers I refer to would substitute the phrases, The Science of Correct Reasoning, or The Science of Valid Thought. But a science which distinguishes between correct and incorrect reasoning—between valid and invalid thought—is just as much the science of one as of the other; and if Logic does not establish the invalidity of all reasoning which does not conform to its rules as conclusively as it proves the

validity of all reasoning which does, the whole Syllogistic theory rests on the *exclusive* validity of the modes of reasoning which it enumerates as legitimate. In giving an analysis of *all* correct reasoning, it is necessarily implied that all reasonings which cannot be thus analyzed are incorrect. Such terms, therefore, as "correct," or "valid" may be thrown off in defining our science, and we may determine what Logic is the science of without regard to them.

In Dr. Murray's Compendium, Logic is defined as The Art of using reason aright in acquiring and communicating knowledge. This definition seems open to many objections. Logic is defined as an art, not as a science. The word "aright," according to the last observation, is superfluous, or rather would become so if the word "science" was substituted for "art." Logic, as I have expounded it (and as Dr. Murray himself expounds it), has no concern with the communication of knowledge except where that communication takes the form of reasoning, and even then it does not require the person who communicates knowledge to others to throw his reasoning into the Syllogistic form. But the main defect of this definition consists in the employment of the word "Reason" without any explanation of what is meant by it: for Reason is used in various senses, both by writers on Psychology and in popular discourse, and in some of these senses the definition is wholly incorrect. Sir William Hamilton defined Logic as The Science of the Necessary Laws of Thought —with the qualifying addition "as Thought." Dean

Mansel's definition is substantially the same, while the authors of the Port Royal Logic defined it as The Art of Thinking. But without here considering what constitutes necessity and law, or what some writers term Form, in mental operations, the question immediately arises, What is Thought? The word, like Reason, has been used in very various senses; and to discover what it means in the mouths of Sir William Hamilton and Dean Mansel, we must study their Psychological systems. One of the Kantian definitions of Logic is, The Science of the Necessary Laws of the Understanding and the Reason. The word Understanding is as ambiguous as Reason, and with many persons the two terms are synonymous. Kant's meaning can only be discovered by studying the peculiar use of these terms in his Philosophy. Mr. Walker, in his Commentary on Murray, defines Logic as The Art of Reasoning, and Archbishop Whately in his well-known treatise adopts the same definition, adding the word Science to Art. But though the word Reasoning is less ambiguous than the word Reason, its meaning is by no means absolutely fixed. Mr. Mill defines Logic as The Science of Proof or Evidence. But what constitutes Proof or Evidence is by no means finally settled. In all these cases, no doubt, the student can discover what the authors intend to treat of (in which they are not always agreed); but this discovery must be made by studying the contents of the treatise, and not by the definition which stands at the head of it.

The truth is, that the difficulty in this respect is

unavoidable. The processes of which Logic treats are mental processes. But as soon as we advance beyond the senses, which have bodily organs appropriated to them, and are thereby readily distinguished, mental processes, and the mental faculties to which they are referable, do not fall into a definite and undisputed number of distinct heads. Psychology, or the Science of Mind, which treats of them, is a Science in which little has been discovered with certainty, and much is disputed. Psychologists differ most widely as to the number and character of the distinct mental processes, and distinct mental faculties, which should be enumerated in their systems, while those who are substantially agreed in their analysis have not arrived at any agreement with respect to the terms to be employed. Mental processes, moreover, from their very nature, have forced themselves upon the attention of the vulgar, and have had names appropriated to them in popular discourse; but, as might have been expected, this popular analysis goes but a very little way, and the terms employed to denote its results are seldom clearly distinguished from each other. Psychologists for the most part took up these popular terms instead of inventing new ones, modifying their meanings in order to suit their own purposes: and thus it has happend that such terms as Thought, Reason, Understanding, Idea, &c., have not only acquired different significations in different Psychological Systems, but have likewise a vague popular signification considerably wider than that in which they are used by any psychological

author. The employment of such terms in a definition of Logic is therefore useless, or worse than useless; and until Psychologists are better agreed, both as to the number and nature of the mental processes, and as to the terms by which they are to be denoted, it seems impossible to put anything better in their place. The Hamiltonian definition of Aristotelian Logic as The Science of The Necessary Laws of Thought, for instance, is of more use in impressing on the mind of the student what Hamilton means by "Necessary Law," and by "Thought," than in explaining to him the nature of Logic; but it cannot perform the former function until the true character of the Science of Logic is understood.

Abandoning, therefore, any attempt at a strict definition of Logic, let it suffice to describe it as the Science which treats of Terms, Propositions, and Arguments or Reasonings, with the corresponding operations of the mind, by whatever names they are to be designated. By Terms I mean such terms (whether consisting of one or of many words) as are capable of becoming the subject or predicate of a Proposition; and by Arguments I mean propositions connected, or alleged to be connected, with each other in such a way that one of them follows from another or others. And by "follows" I mean follows conclusively — follows independently of the particular terms employed (assuming the propositions in question to have been reduced to the forms required by Logic; for in reducing them to these forms we must often direct our attention to the meanings of

the terms), and would equally follow if the actual terms were replaced by the letters of the alphabet. In short, Logic can only recognise as correct those inferences which are implied by the relations which are *asserted* to exist between the terms. It takes no notice of any *unexpressed* relations which we either know or believe to exist between them. They must be expressed before the reasoning is made to turn on them. The inference, for example, from *nearly all* to *all*, which is frequently made in Physical Science, is not recognised by Logic, because it is not true of all classes of things, that what belongs to nearly all of them belongs to all. If we have any special reasons for believing that this is true of the particular class of things we are dealing with, Logic requires these reasons to be stated. The best logical way of stating such an argument, however, is in the hypothetical form, viz.:

> If this property belongs to nearly all the Bs, it belongs to all the Bs;
>
> This property belongs to nearly all the Bs;
>
> ∴ It belongs to all the Bs.

Here the reasoning is conclusive: but the probability of the hypothetical premiss will depend on the nature of the property in question, and the meaning of the term B, and will be different in each case. Supposing the truth of the categorical premiss to be certain, the probability of the conclusion will in all instances

be the same as that of the hypothetical premiss. I may, however, notice that the number of instances from which universal conclusions are drawn in Physics often fall considerably short of the Nearly-all, though I doubt if it ever descends to the bare indefinite *Some* of Logic.*

The same difficulties which are experienced when we attempt to define Logic itself are also encountered when we seek to define many of its principal terms. The meaning in which I have used them will, I think, be sufficiently clear from the preceding pages, and that is all that I desire. Definitions do not form the groundwork of Logic, or of any other science, and are only useful in explaining the terms employed. If these terms are sufficiently understood without definitions, the scientific writer need not define them. This follows from what has been already laid down. Definitions are always Analytical propositions; but the important part of every Science is the Synthetical propositions which it contains; and it is, I think, sufficiently evident from the foregoing pages, that from no number of Analytical propositions can a Synthetical proposition be logically

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* How should the proposition Nearly all Bs are Cs be treated in Logic? Most writers would say either that we are to drop the "Nearly," and treat it by a sort of courtesy as an universal proposition, or else since *Nearly-all* falls short of *All* we must cut it down to the bare logical *Some*. I think it better to treat *Nearly-all-B* as a distinct term from B—a sub-class of the Bs nearly, but not quite, coincident in its extension with the higher class B. The proposition is therefore an universal proposition, but its subject is not B, but Nearly-all-Bs.

deduced. We might as well attempt to arrive at the number 5 by the perpetual addition of 0s.*

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* Do Scientific definitions, then, merely explain the meaning of the terms employed? They sometimes do more than this; but in such cases they will be found to contain more than mere definitions, and the writer accordingly deems it necessary to prove them as he goes on. Thus, when a scientific writer defines a term already in use, he very often means to imply that the extension of this term—the collection of objects which it denotes—is the same as that which the same term has in its popular use. But to assert that this is the case is not to define, but to make a synthetical proposition. Every B is B becomes a synthetical proposition, if the predicate B means that term as popularly used, and the subject B the same term as used in a special scientific sense or *vice versâ*. What constitutes an identical proposition (the most futile kind of analytical propositions) is not that the subject and predicate are identical in sound, but that they are identical in meaning. However, it is not universally true that the extension of a scientific term is the same as that of the same term in its ordinary use; and in no scientific treatise can such a coincidence be assumed without proof—including under proof an appeal to experience.

Mr. Mill thinks the material element of scientific definitions is, that they assume the *existence* of corresponding objects, and he contends that what appears to follow from the definition really follows from this assumption. But this doctrine cannot be admitted. The assumption that a class of things exists, whose attributes are those connoted by a general name, cannot logically lead to the conclusion that this class of things possesses any *other* attribute not connoted by that name—except of course the attribute of existence. The existence of a thing possessing certain attributes must not be confounded with the co-existence of these attributes with others, and Logic will not enable us to bridge over the chasm which separates simple existence from co-existence.

The observations in the text may perhaps lead the reader to object that the Dictum and the Axioms being Analytical propositions cannot form the basis of the Science of Logic. This objection is well-founded. The real basis of Logic is not the Dictum or the Axioms as such, but the Synthetical proposition that all valid reasonings can be brought under them. If Aristotle had only discovered that the Dictum *was true*, no one would have gained much by the discovery.

CHAPTER XII.

FURTHER REMARKS ON TERMS AND THEIR MEANINGS.

Treatises on Logic usually contain divisions of Terms, many of which have little or no connexion with the Science, but are not useless for other purposes. Thus terms are divided into Categorematic, Syncategorematic, and Mixed. A Categorematic term is one capable of being employed by itself as the subject or predicate of a Categorical proposition. A Syncategorematic term can only be employed in this way when conjoined with others, while a Mixed term consists of a Categorematic term and a Syncategorematic term in conjunction. A Mixed term, as Whately observes, is really Categorematic, consisting of what has been called a many-worded name; and it seems best to regard Syncategorematic terms, not as terms, but as parts of terms. Another division of Terms is, into Univocal, Equivocal, and Analogous. An Equivocal term is one which is employed in more than one sense, and so is an Analogous term, with the addition that in this case there must be some analogy between its several meanings. What constitutes a term, however, is not the sound but the meaning, and therefore all terms should be treated in Logic as Univocal. If the same sound is employed with two distinct meanings, it should be regarded as two distinct terms. Such ambiguous words, however, as has been already remarked, are among the most ordinary sources of fal-

lacies; and when treating of fallacies their existence must, of course, be recognised by Logicians.

The distinction between General, Collective, and Singular terms has been already noticed, and it has been remarked that Collectives may be treated as Singulars. The main distinction between these classes of propositions for logical purposes is, that Singulars and Collectives do not admit of being used as the subjects of Particular propositions, and are very rarely used as predicates. This last circumstance results from the fact that singular terms seldom have any definite connotation. The attributes by which John Smith is known to one of his acquaintances are not those by which he is known to another, and his attributes will, moreover, be different at different periods of his life. The term John Smith, therefore, does not connote or imply any fixed and definite collection of attributes like the terms Man and Horse. Some of John Smith's attributes indeed remain unaltered, such as, for example, that he was born in Londonderry, and that his father was a Town Councillor; but many of his acquaintances are ignorant of these attributes, which cannot therefore be implied or connoted by the name.

Another division of terms is into Abstract and Concrete. An Abstract term is a term which denotes an Attribute or collection of Attributes, while a Concrete term denotes a Thing or a number of Things, to which certain attributes belong. The words Abstract and Concrete are, however, usually employed as relative terms, every Abstract term having its corresponding Concrete,

and *vice versâ:* according to which view Proper Names, though denoting things, would not be called Concrete terms since there are no corresponding Abstract terms. This arises from the fact already noticed, that Proper Names have no definite or fixed connotation. John Smith, in this way of speaking, is not a Concrete term, because there is no such attribute or collection of attributes as John Smith-ness; but just (or just-things) is a Concrete term, because it means all things which possess the attribute denoted by the corresponding Abstract term justice. Abstract terms would seem to be singular terms when considered logically, but the propositions in which they occur can generally be turned into Universal propositions, by substituting for the Abstract term the corresponding Concrete. Thus the proposition Justice is a virtue, is the equivalent of the proposition All just actions are virtuous, or rather it would be so if the terms Justice and Just, Virtue and Virtuous, *exactly* corresponded. But in turning propositions containing Abstract terms into propositions containing the corresponding Concretes, we must take care to avoid falling into what Archbishop Whately calls the fallacy of Paronymous Words. In the proposition Justice is a virtue, we consider justice as a principle in the mind; but by just actions we often mean not actions resulting from this principle, but actions which are such as this principle would dictate, though in point of fact they may have proceeded from some other principle. But the proposition All just actions are virtuous, is only the equivalent of the proposition Justice is a virtue, if

by just actions we mean actions proceeding from the principle of justice in the mind.

Another division of Names is into Relative and Absolute or Non-relative. Some writers speak of all or almost all names as relative, but the distinction in question appears to be as follows:—A Relative name necessarily brings before our minds some object or objects which it does not denote, while an Absolute name does not necessarily call up any other objects than those denoted by it. Thus, the word Son necessarily calls up the notion of Parent, though denoting the child only, and the idea of a subject necessarily calls up the idea of a ruler though it does not denote him. On the other hand, the word Man does not necessarily call up anything more than the individuals or some of the individuals which it denotes. Some writers indeed would say that it necessarily calls up the idea of That which is not a man, or Not-man. Whether this is the case it is not necessary to inquire. If it be so, we should describe a relative term as one which necessarily calls up, in addition to its own denotation, something more—something more specific—than the mere negation of itself or of its denotation; while a term which calls up nothing but its own denotation, together with the corresponding negative, should be called Absolute. Relative names, it may be noticed, may be Abstract as well as Concrete. An abstract relative name is the name of the relation itself considered as an attribute, and the peculiarity which this attribute possesses is that it necessarily belongs to pairs or larger numbers

of things, not to single things. Neither sovereignty nor paternity could be an attribute of a single thing existing by itself. The former supposes subjects as well as a sovereign to rule over them—the latter a child as well as a father. The attribute or relation is not complete without both. On the other hand, snow would be white although the whole surface of the earth was covered with it. The attribute white belongs to it independently of anything else, unless indeed the white supposes the not-white.

Another division of terms is into Positive, Privative, and Negative. A Positive term implies the presence of one or more attributes in the Things denoted by it; a Negative term implies the absence of one or more attributes in the Things denoted, while a Privative term implies the presence of some attributes and the absence of others which are usually found along with the former. But in this exposition, all attributes must be regarded as positive, whereas we often hear of negative attributes; and the Logician cannot lay down any test (or at least Logicians are not agreed as to the existence of any test) by which positive attributes or positive Things can be distinguished from negative ones. It may be better, therefore, to define a Negative term as a term negative in form (*i. e.* a term in which "non" "un" "in" "mis," or some other negative particle occurs); and a Privative term as one privative in form (often indicated by such particles as "ex" "ab" "de"); while all other terms should be treated as Positive.

A similar distinction to this last, but much more apposite to the purposes of Logic, is the distribution of terms in pairs, as Contradictory, Contrary, and what may be called Congruent. Contradictory terms (after the analogy of Contradictory propositions) are such that both cannot at once be predicable of the same subject, while one must be predicable of any subject whatever. They are always of the form B and not-B. Contrary terms (after the analogy of Contrary propositions) are such that both cannot at once be predicable of the same subject, while there may be subjects of which neither is predicable. Congruent terms are such that both may be predicable of the same subject, or in other words, such that the connotations of both may co-exist in the same thing. Some writers confine the phrase Contrary terms to the two sub-denominations of the same class which are most opposed to each other. The analogy to Contrary propositions is thus lost sight of, and it is moreover not true that every class of things contains two such sub-denominations. The writers I allude to would give such instances as Black and White, the two most opposed sub-classes of the class Colour. But in the first place it seems doubtful whether Black (which is the mere absence of light) should be reckoned as a colour; and, secondly, if so reckoned, the absence of light is as much opposed to light of any other colour as to White light.

Besides classifying Terms or Names, Philosophers have classified Things, and of course any classification of things may be considered as involving a similar classi-

fication of the names or ideas of them. This, however, is a question with which the Logician is not concerned; and, moreover, no classification of things has been universally accepted as the best; and indeed it will be impossible to classify things in the best way until we know more about them than at present. A classification known as the Predicaments or Categories, however, has figured largely among the writings of Logicians, though there is some dispute as to whether it was meant as a classification of Things or of what might be asserted about Things. The original list as given by Aristotle (whether he regarded it as complete or not) was as follows:—*οὐσία, ποσὸν, ποιὸν, πρός τι, ποιεῖν, πάσχειν, ποῦ, ποτὲ, ἔχειν,* and *κεῖσθαι.* This classification seems to have been founded on the parts of speech which were recognised by the grammarians of the day. The terms were rendered into Latin by—*Substantia, Quantitas, Qualitas, Relatio, Actio, Passio, Ubi, Quando, Situs,* and *Habitus;* and into English by—Substance, Quantity, Quality, Relation, Action, Passion, Where, How Long (or Space, Time), Posture, and Habit; each of which heads was variously subdivided. On looking over the list, it will be seen that the first head alone stands for things properly so called, while all the others stand for attributes or properties of things; and, therefore, it would seem as if the first division or classification should have been into Substances and Accidents or Attributes, which latter term might have been subdivided into the nine subsequent heads. Again, the last six heads appear to be

various kinds of relations or relatives, and should therefore have been described as sub-divisions of the fourth; and why all relations should be subdivided into these six, and no others, it is not easy to explain. No writer on Logic, however, would probably defend this as even the best extant classification of Things and their properties, and there is, therefore, no reason why it should be retained. Mr. Mill proposes to substitute the following: Feelings or States of Consciousness, Minds, Bodies, and the Elementary Relations among Feelings or States of Consciousness, namely (according to him), Similarity, Succession, and Co-existence.* No Logician could venture to adopt such a classification without entering upon a long discussion to justify it. Why, for example, should not the elementary relations among our States of Consciousness be described as States of Consciousness themselves? or why do we class the relations of similarity, succession, and co-existence as elementary, while excluding all other relations as composite or derived? Mr. Mill's classification is, in fact, founded on a Psychological theory which

* Elsewhere, in analysing the Import of Propositions, Mr. Mill classes what can be asserted about things as follows:—Existence, Co-existence, Sequence, Causation, and Resemblance. This list agrees with the former, if we recollect that Minds and Bodies are Things, and not what can be asserted about Things—that Existence consists, according to Mr. Mill, in Causing Feelings of some kind, while Causation itself is only immediate and invariable Sequence or Succession. But, then, does not Mr. Mill, in his Categories, mix up Things with what can be asserted about them? Similarity, Succession, and Co-existence, at least, seem to fall under the latter head; and there is clearly no difference between Succession and Sequence, or between Similarity and Resemblance.

has not met with universal acceptance. The Kantian Categories, on the other hand, are not classifications of Things, though in a certain sense they are classifications of what may be asserted about things. Every assertion about things, says Kant, is made by a proposition or judgment: if, therefore, I can arrive at a complete classification of propositions or judgments, I can arrive at a complete classification of what can be asserted about things, or at a complete list of the Categories. Though this hardly falls within the scope of the present chapter, I shall here give Kant's results:—Judgments or Propositions differ in four respects, namely, in respect of Quantity, Quality, Relation, and Modality. (What is meant by the two latter terms will be best understood by their sub-divisions.) Hence the four leading Categories or Predicaments are Quantity, Quality, Relation, and Modality. Each of these is sub-divided into three heads. In respect of Quantity, propositions or judgments are divisible into Universal, Particular, and Singular. It may be objected to this, that in Logic Singulars need seldom be distinguished from Universals; but in the Kantian Philosophy the principal use of the Categories is in Psychology or Metaphysics, not in Logic. In respect of Quality, propositions are divided into Affirmative, Negative, and what he terms Infinite—that is, propositions affirmative in form, but negative in meaning, such as, The defenders of the fortress were unprepared. So far as Logic is concerned, there seems to be no reason for not treating such propositions as affirmative. In respect of Relation, pro-

positions are divided into Categorical, Hypothetical, and Disjunctive; and in respect of Modality, into Problematical, Assertorial, and Apodictical, that is, Propositions of the respective forms B *may be* C, B *is* C, and B *must be* C. The first and third of these latter forms I have not recognised, but only because Syllogisms in which they occur follow the same rules as if the copula was the simple *is*, and rare, moreover, of are occurrence in practice. From the propositions All B may be C, and All C may be D, we can infer All B may be D, in exactly the same way as if "is" had been substituted for "may be" in each premiss; and a similar observation will apply to "must be."* To these subdivisions correspond the sub-categories in the Kantian list. Under the Category of Quantity he places the Categories (sub-categories) of Unity, Plurality and Totality; under Quality, Reality, Negation and Limitation; under Relation, Substance, Causality, and Community or Reciprocal Action; and under Modality, Possibility, Existence, and Necessity. The Kantian Categories have this advantage, that it does not require very extensive knowledge or erudition to classify Propositions in the best way, nor do they admit of being classified in so many different ways as Things. But it may be doubted whether Kant's classification is the best, at least for

* But if "may be" occurs as the copula in one of the premisses, the copula in the conclusion will only be "may be"; and to warrant a "must be" in the conclusion, a "must be" should have occurred in *both* premisses. No chain of reasoning is stronger than its weakest link.

Logical purposes; and other Philosophers have endeavoured to reduce his heads to a smaller number. Thus Dean Mansel recognises none but Unity, Plurality, and Totality, while M. Cousin tries to reduce the entire list to Substance and Causality. The Categories of Relation show clearly that Kant recognised Hypothetical and Disjunctive Judgments as not reducible to the Categorical form; but his divisions of Judgments into Universal, Particular, and Singular, into Affirmative, Negative, and Infinite, and into Problematical, Assertorial, and Apodictical, are, in fact, divisions of Categorical Judgments only. If Hypotheticals are regarded as irreducible to the Categorical form, it is impossible, for instance, to describe the proposition *If Newton is right, all bodies in the solar system move under the force of gravity*, as either an universal or a singular proposition, for the antecedent is singular and the consequent is universal. If, on the other hand, the mode of reduction already given be recognised, all Hypothetical and Disjunctive propositions are affirmatives, and, indeed, Universal (or Singular) affirmatives; and the Kantian division, in respect to Quality, and even Quantity, is thus excluded, so far as they are concerned. The Kantian Categories, therefore, can hardly be admitted into a Treatise on Logic which seeks to deal only with what is certain and indisputable.

The Predicables of the earlier Logicians are not to be confounded with the Categories or Predicaments. They were divisions not of Things, but of what might be asserted about Things; or, more accurately, of what

a predicate could *affirm* about a subject. According to Aristotle, they are four in number, namely, Genus, Definition, Property, and Accident. The predicate of an affirmative Categorical Judgment, according to him, must be either the Genus (or higher class) under which the subject was ranked, the Definition of the subject (a class coincident with it), or it must express some Property or Accident of the subject—a Property meaning an attribute belonging to it necessarily;* while an Accident, as implied by its name, was an attribute which it might or might not have. The chief objection to this list seems to be, that the Logician has no means of knowing whether an attribute alleged to belong to a subject belongs to it necessarily or accidentally—or, indeed, belongs to it at all; for the allegation that it does so may turn out to be untrue. If by an attribute necessarily belonging to a subject we mean one con-

* According to some, Aristotle meant by Property an attribute belonging to the subject not only necessarily, but *exclusively*, so that when a Property was predicated, as in the case of a Definition, the predicate and subject were co-extensive; while if the Genus or Accident was predicated, the predicate included the subject in its extension, or was the Whole, of which the subject formed a Part. The Aristotelian Predicables on this view are reducible to two. The predicate was, in ~~Dean Mansel~~'s language, either Substitutive or Attributive, the real quantity of the predicate (of an affirmative proposition) being in the former case universal, and in the latter particular; thus to a large extent anticipating the theory of Sir William Hamilton, which will be noticed hereafter. But the predication of a Definition or Property means the predication of something which is *in fact* a Definition or a Property of the subject, not the *assertion* that some predicate is a Definition or a Property of the subject. Whether C was the Genus or the Definition of B, the proposition

Archbishop Thomson's

noted by the name of the subject, Property will be found to be identical with Genus, or sometimes with Definition. If not, how is the necessity or want of necessity to be ascertained? For logical purposes, it is much better to adopt Kant's simpler division of propositions into Analytical and Synthetical. Those in which the Genus or Definition* of the subject is predicated of it will be Analytical Propositions: those in which an Accident is predicated will be Synthetical, while the term Property which involves an ambiguity should be avoided. Aristotle's followers afterwards affirmed that nothing could be defined except a species (or sub-class), and that the proper mode of defining it was by its Genus and Essential Difference. This doctrine led them to reject Aristotle's head Definition, and to substitute for it the two heads, Species and Differentia. This alteration can hardly be considered an improve-

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asserting the relation between them would have been equally expressed in Aristotle's System, as All B is C; nor did he or his followers ever allege that in the one case the copula "is" *meant* (in the Dean's words), "constitutes," and in the other "is contained under." Whether, therefore, the Hamiltonian theory of the Quantification of the Predicate is sound or otherwise, it was not anticipated by Aristotle's Predicables, much less by the Predicables subsequently adopted by his followers. None of the Aristotelians ever thought of *expressing* a Proposition in which a Property of the thing was asserted in the form All B is *all* C, or All B *constitutes* C. It may be added, that it does not seem to be the fact that every class of things has some attribute which belongs to it exclusively, if that was what Aristotle meant by *ἴδιον* or Property. What belongs to it exclusively is, generally speaking, the entire collection of attributes connected by the class-name.

* Except perhaps in a Singular Judgment or Proposition.

ment. What is the difference between predicating of any given subject its Genus and its Species? The Logicians I refer to would describe the proposition *John Thomson is an animal* as one in which we predicate of John Thomson the Genus to which he belongs, while the proposition *John Thomson is a man* predicates the Species. What then becomes of the proposition *John Thomson is an Englishman?* Is not Englishman a Species of which man is the Genus, and, therefore, should not the proposition John Thomson is a man be therefore regarded as one in which the Genus is predicated also? Again, if the proposition John Thomson is an Englishman is regarded as one in which the Species is predicated, how are we to classify such propositions as John Thomson is a poor Englishman, or John Thomson is an honest Englishman? Are not poor-Englishman and honest-Englishman again sub-classes, or Species, of which Englishman is the Genus? And as we may go on subdividing a class to any extent, the sub-classes being still general terms (consisting usually of many-worded names), it is impossible to arrive at any Species which may not be converted into a Genus by further subdivision. Predicating the Genus and the Species of any given subject then comes to exactly the same thing: and if the subject be a class or a general name (as is usually the case), both predications equally consist of bringing that class under a larger class. The proposition, Negroes are men, does this quite as effectually as, Negroes are animals, or Negroes are organized beings: and all three are

Analytical propositions to everyone who understands the meaning of the term Negro. Nor is Essential Difference of greater practical utility. What is the Essential Difference between the class Animal and the sub-class Dog, or between the class Matter and the sub-class Gold? If by Essential Difference we mean the difference between our ideas of the two things—between the connotations of the two names—we shall find that in most cases it does not consist of any one attribute, but of several taken in conjunction; while if we mean by Essential Difference the Essential Difference between the Things denoted by the names, we shall find, by a reference to Natural History or Chemistry, that there are usually a great number of known differences, all of which appear to be equally essential or unessential. Again, what is the distinction between an Essential Difference and a Property? The Logician would say that a Property is not the principal difference, but one resulting from it. But when he is called upon to prove that an alleged Property results from the alleged Essential Difference, he must in most cases confess his inability to do so. If the weight of Gold be its Essential Difference, how does the colour result from the weight? or if the colour be the Essential Difference, how does the weight follow? Moreover, the moment we begin to inquire into the nature of Genus, Species, and Essential Differences, we find ourselves investigating the properties of Things, instead of examining the character of the reasoning process. This knowledge of Things is not, and probably never will be, complete; and if we

had to wait until Physicists had arrived at a complete theory as to the nature of the Genus, Species, Differences, Properties, and Accidents of Natural Objects before constructing a Science of Logic, the completion of that Science would be indefinitely postponed.

It would be easy indeed to retain a good many of these terms with such changes in their meanings as would reconcile them with the view of Logic adopted in the present work. But no amount of alteration would make them relevant to the subject treated of, and there is nothing which has so much retarded the progress of Logic as the introduction of irrelevant matter. In connexion with these Predicables, Logicians have introduced a distinction between Necessary and Contingent Matter, which is sometimes expanded into Necessary, Impossible, Possible, and Contingent. Now, the ablest physical investigators have hitherto failed to discover any test for distinguishing what is necessary and what is contingent in the external world: and to suppose every student of Logic to know what is necessary and what is contingent is almost to suppose him possessed of Omniscience. And, after all, what is the use of the distinction? It enables us to quantify the subjects of some propositions where they have been left unquantified, and to state some of the rules of Opposition in a short form. I will take Dr. Murray's examples of the former. The proposition Angels are incorporeal means *All* angels are incorporeal, because that is necessary matter, but the proposition Soldiers fortify camps means *Some* soldiers fortify camps, because the matter is contingent. First, then,

how is it proved that angels are necessarily incorporeal? and secondly, even if they are so, how does it appear that the propounder of the proposition before us meant to assert it?—for the question is not what he might have said with truth, but what he in fact intended to say. Or suppose he asserted that Angels are *not* incorporeal, how are we to quantify this proposition? The matter being necessary according to Dr. Murray, the assertion is equally untrue whether he meant All angels or Some angels, and, therefore, our distinction leaves us no wiser than we were before. In any case of the kind you will find it much better to consult the context than to have recourse to this illogical distinction.

The inquiry, what it is that a General Term stands for, seems to have a closer relation to Logic. The answer has been given already. Every general term connotes an attribute or collection of attributes, and denotes all the objects in which that attribute or collection of attributes is to be found. It may be said to stand for the one or the other, according to the meaning we attach to the phrase "stand for." The question is sometimes stated, 'What is the mental state which corresponds to a General Term?' or expressed in some similar form of words. But many Philosophers have maintained that the connotation of a General Term cannot be realised in the mind entirely apart from its denotation, and that the only way in which we can realise an attribute or collection of attributes is by contemplating them in an object present to sense or imagination: in which case they are always accompanied by other attributes characteristic of

that individual object. Logic need not concern itself with the truth or falsity of this doctrine. It is sufficient for our purpose if we can distinguish the attributes connoted by the General Name from the others which are simultaneously present, and regard the former alone as forming a distinct collection. Some writers, who admit that this can be done, affirm that it can only be done by the mediation of language — that we can only contemplate this collection as a distinct one by the aid of the name which connotes it. It rather appears to me that the General Name cannot come to connote a distinct collection of attributes until we have already made that collection a subject of separate consideration. But for the purpose of a treatise on Logic the inquiry is immaterial, since a writer can only deal with such collections of attributes as are connoted by names—including, of course, what have been called many-worded names. The reader will now understand what Archbishop Whately means by defining a General Notion as "an inadequate notion of an individual." It can, he thinks, only be realised in the mind by calling up the notion of an individual, and then attending to a part of that notion only, neglecting the rest. But the definition is not a good one. The important point is not what the General Notion excludes, but what it includes. Its essence does not consist in its inadequacy to represent the individual, but in its adequacy to represent the general. There may be a hundred different ways of forming an inadequate notion of the individual, but of these one only will answer our purpose. I may

form any number of inadequate notions of John Thomson by attending to some of his attributes to the neglect of others; but the term Man connotes a definite number of these attributes, and the General Notion of Man is formed by attending to this definite collection, taking care neither to add to nor to subtract from it. Without this explanation the doctrine of Whately and other Nominalists is calculated to mislead: and it is sometimes exaggerated into an allegation that there are no General Notions, and that nothing is general except names—whence it is inferred that all general reasonings deal with names only. But Notions can plainly have the same generality that Names have. Considered as a mere sound, the Name is an individual thing, and a different thing each time it is spoken or written. Its generality consists solely in its signification. So the idea of Man in my mind at this moment is an individual thing, and a different thing from the idea of Man which was there half an hour ago: but then it is equally general in its signification. And, in truth, the Name derives its signification, and therefore its generality, from the Idea. Take away the Idea of Man (such as it is) from the human mind, and the Name Man sinks to a mere sound, losing all significance, and therefore all general significance. Nay, even to regard the Name Man—that is the sound Man—as always constituting *the same* Name or sound we require a General Idea, and there is no reason for supposing that it is easier to form a General Notion of a sound (or rather of sounds) than of many other classes of things.

CHAPTER XIII.

ANALYTICAL AND SYNTHETICAL PROPOSITIONS.

WHILE the distinction between Analytical and Synthetical Judgments or Propositions has been overlooked by some writers, its validity has been disputed by others; and others again, by whom the distinction is admitted, have laid down theories wholly subversive of it. It therefore becomes necessary to examine the distinction at greater length than would otherwise be necessary. It is perhaps desirable in the first place to inquire briefly whether terms (or words) stand for Things or for our Ideas of Things—a question which some Logicians regard as of great importance. The dispute would probably have been put an end to by settling, in the first place, what is meant by Things, and what by Ideas; and in the next place, what is meant by the phrase "stand for." Some of those who insist most strongly that words stand for Things, tell us at the same time that the only Things of which we can know or affirm anything consist of collections of our Sensations or other Mental States, which Sensations or Mental States would probably be designated Ideas by their opponents; and between such disputants the question is mainly one of words. This dispute is practically concerned with general terms only; and the principles already laid down seem sufficient to decide it. General terms have both a comprehen-

sion and an extension—a connotation and a denotation. The comprehension or connotation is mental, and would by most writers be termed an Idea; while the extension or denotation is a collection of objects, which would be called Things. Of these two elements we have seen that the comprehension or connotation is the more important, and is in fact that which determines the other. The most correct statement would thus seem to be, that general terms stand for Things which correspond to our Ideas. If there is anything that corresponds to the idea—that is to the connotation or comprehension—the general term stands for that thing; and if there is anything that does not correspond to it, the general term does not stand for that thing—that is if the phrase "stand for" means "denote." But as this phrase is ambiguous, it is better to drop it altogether, and to say that General Terms connote Ideas, and denote the Things which correspond to those Ideas.

We are now in a position to see how far it can be admitted that Judgments or Propositions consist in, or result from, comparing our ideas and recognising that one is contained in, or not contained in, another; or that they consist in recognising the agreement or disagreement of our ideas—a doctrine which may be variously expressed. In an affirmative Analytical Judgment or Proposition, the connotation of the predicate forms a part (or the whole) of the connotation of the subject; and it may therefore be truly said, that in such a judgment or proposition we compare

our ideas, and ascertain that one is contained in the other. In the negative Analytical, on the other hand, the connotation, or some part of the connotation of the predicate contradicts some part of the connotation of the subject; and therefore mere comparison of the ideas (or connotations) suffices to show their incompatibility. But in the case of a Synthetical Judgment or Proposition, no comparison of the ideas (that is of the connotations) is sufficient to enable us to decide the question. The agreement or disagreement is not to be found in the comprehensions of the subject and predicate, but in their extensions — that is in the Things. An affirmative Synthetical Proposition does not assert that two Ideas agree, but that the Things corresponding to those Ideas agree, or rather that either the whole or some parts of them are identical; for the word "agree" tells us little until the precise kind of agreement is ascertained. I might contemplate or compare my Ideas of an animal with horns on the skull and of an animal that ruminates (or chews the cud) for ever without ascertaining whether they agree or disagree with each other: but when I come to examine the corresponding objects in nature, I find that, as a matter of fact, every animal which possesses the former property possesses the latter also. This fact I could never have discovered by comparing the connotations of the terms, nor when discovered does it make any difference in these connotations. What it does is to bring the denotations—the extensions—into a relation which I did not know before. If,

therefore, by ideas we mean connotations, it is not correct to say that the proposition, All animals with horns on the skull are ruminants, expresses any comparison, relation, or agreement, between Ideas. The agreement or relation exists only between the corresponding Things.*

The grounds on which the validity of the distinction between Analytical and Synthetical Propositions has been disputed seem to be two-fold. First it is alleged that the formation of the synthetical judgment only requires a deeper or more careful scrutiny of our ideas, bringing out elements in them which are not obvious on first inspection. To this it may be replied, that our general notions are of our own making, and they contain no more than what we put into them. If I do not put the idea of rumination into the notion which I form of an animal with horns on the skull, it forms no part of my idea of such an animal, and no amount of analysis or comparison will enable me to discover it

* The doctrine which I am contending against often takes the shape of describing the relation between the Subject and Predicate as that of Whole and Part—every affirmative proposition asserting that the Predicate is a Part of the Subject. If it is meant that the connotation of the predicate is part of the connotation of the subject, this statement is only true of Analytical Propositions. If it is meant that the denotation of the subject is a part of the denotation of the predicate (or rather is asserted to be so), this is true in the case of an universal affirmative (A), though the existence of such a relation cannot be ascertained by mere comparison of Ideas or connotations. But a particular affirmative (I) merely asserts the identity of part of the denotation of the one with part of the denotation of the other. It leaves it quite undetermined which has the larger denotation. Consequently no relation of Whole and Part is asserted in it.

therein. The writers I refer to seem to be rather speaking of a sort of Standard Idea which is not alleged to exist anywhere, but which according to them *ought* to exist, and of which this property *ought* to form an element. I *ought*, it is contended, to put into my idea of a horned-animal every property which all horned animals possess, and as rumination is one of these properties, my idea of a horned animal is incomplete until I have made this addition to it; while of course, after I have made it, the property can be discovered in the idea by mere analysis. The answer to this is, that there is no such Standard Idea; nor is there any reason why I should introduce into the comprehension of every general term every property which belongs to the individuals included in its denotation. On the contrary, it is frequently convenient to consider objects from a partial or abstract point of view—to consider some of their properties without considering the others—in order to accomplish which it is necessary to separate in thought what exists together in nature. Indeed without this separation we could not form general notions at all. My idea of an animal horned on the skull is equally complete as it stands, whether it be true or false that all such animals ruminate. There is no reason why I should include rumination in it in either case; and to do so would be to subvert all etymology, to render dictionaries useless after a short time and to make an intelligent study of the writings of earlier authors impossible. A language in which the *term* Horned-animal had come to *mean* Horned-ruminating-animal—

in which all the known properties of the Things corresponding to every term had come to form part of the meaning of that term — would soon become equally impossible to learn and to speak. No man in fact could speak it with accuracy, or fully understand it, until he had mastered and fixed in his memory every known property of every known class of objects, and even then the meaning of the terms would change with every new discovery.

Nor is this all. Let us suppose, if possible, our present idea of a horned-animal obliterated, and that of a horned-ruminating-animal installed in its place. Would this latter idea afford us the knowledge which is now conveyed by the proposition, Every horned-animal ruminates? Certainly not. Our ideas do not prove the existence of corresponding things, or else it would be easy to prove the existence of dragons, ghosts, and unicorns. Nor do complex (or compound) ideas prove the co-existence of the combined properties. Men exist and green objects exist; but the idea of a green-man does not prove his existence. Suppose, however, that this difficulty was also surmounted, and that we had reached that state of knowledge, and that torpidity of imagination, in which all our ideas corresponded accurately with Things; and that in this state we found ourselves in possession of the idea of a horned-ruminating-animal; how could we infer from such an idea, that whenever the horns were present rumination would be found to accompany them? We should be equally possessed of the idea of a two-footed-

rational-animal. Could we conclude from that idea, that wherever the two feet were found rationality would be discovered along with them? The truth is, that even in this state the presence of the idea would only prove that the attributes comprised in its comprehension were *sometimes* conjoined, not that they always were so; still less (if we designate these attributes by the letters of the alphabet *a*, *b*, *c*, &c.) could we infer from the presence of the idea *a b c* in the mind, that *a* was always conjoined with the united *b c*, but not always with either *b* or *c* separately, while on the other hand *b c* was not invariably conjoined with *a*. But these latter are the kinds of relations which are asserted in Synthetical propositions; and it seems to me impossible to suppose our ideas to have attained a state in which such relations could be discovered in them by mere analysis—meaning of course, by idea, that in the mind which corresponds to a term, and not that which corresponds to a proposition: for the meaning of the word idea is still unsettled.

These observations also dispose of the second objection, which is that Synthetical Propositions are only Analytical Propositions "in the making," as it has been expressed—that as soon as the relation between the subject and the predicate has come to be an object of popular knowledge or belief, the connotation of the term which forms the subject—the idea corresponding to the subject-term—comes to be so extended as to include the predicate, and the proposition then becomes Analytical. But in the first place, this

is not invariably the case. When the known properties of a Thing (or rather of a class of Things) are very numerous, we seldom include the whole of them in the connotation of the class-name; and in other cases the etymology or composition of the name (especially if it is a many-worded name, such as Animal-with-horns-on-the-skull), preserves it from undergoing any change of meaning as our knowledge of the subject-matter advances. It is true, however, that words frequently change their meanings, and that they often do so by tacking on to their former connotation some property which has been found to belong to all the objects which they denote. But when the connotation of a word is altered, that word is no longer the same term, nor is any proposition into which it enters as subject or predicate the same proposition as before. It is not then true, that when the meaning of the term which once formed the subject of a Synthetical Proposition has undergone this change, the Synthetical Proposition in question has become an Analytical Proposition. It is only true that in consequence of a change in the meaning of the terms, the same words which once expressed a Synthetical Proposition have now come to express an Analytical one:* but what

* In this way, as has been noticed by Mr. Mill and others, valuable portions of our knowledge are sometimes lost by gradual changes in the meaning of words. It is by means of propositions that knowledge is transmitted from one generation to another; and nothing is more common than to leave this gradual transition in the meaning of terms unnoticed, and to transmit to the next generation not the same proposition, but merely the same words, which ultimately come to convey something

constitutes the identity of a proposition is not the identity of the words, but the identity of the facts intended to be asserted. To take an example:—There are few facts more familiar to us than the mortality of all mankind. No one can doubt that this assertion, when first made, was a Synthetical Judgment, and one which conveyed important, though unpleasant, information to the hearer. And as a matter of fact, I do not think the word Man has come to connote or *mean* mortality, and therefore the proposition All men are mortal is still a Synthetical Judgment. But let us suppose the contrary. Let us suppose that mortality now forms part of the idea of Man, or of the connotation of the term Man. Does the proposition All men are mortal now convey the same assertion as before? Clearly not. It formerly meant that wherever we met with an animal possessing the attribute of rationality, and the peculiarities of external form which *then* constituted the connotation of the term Man, we should find the attribute mortality along with them. It now merely means that, wherever we find a rational animal possessing these peculiarities of external form, *together with the attribute mortality*, this latter attribute is to be found. It, in fact, makes the same

different and perhaps exceedingly trivial; while the truth formerly conveyed is lost, until some one discovers that our ancestors meant something quite different by the words in question from what we do. The principal causes of this loss of knowledge are, either that the predicate drops a part of its connotation, and so comes to connote nothing more than what is already included in the connotation of the subject, or else that the subject takes on an additional connotation, with which addition

assertion that would formerly have been made by the proposition All *mortal* men are mortal. Nor would such a proposition be of the least use to us in arguing against a pretender to immortality. He would simply say, If by the term Man you mean a rational animal possessing certain peculiarities of form *who is also mortal*, I admit that All men are mortal, but I deny that I am a Man. Neither would there be any difficulty in expressing what was formerly meant by the proposition, All men are mortal, under these changed conditions of language. We should now say, Wherever we meet with the *other* attributes connoted by the term Man, we shall find the attribute mortality along with them. In truth, an Analytical Judgment affirms the identity (total or partial) of the attributes comprised in the connotations of two terms; while a Synthetical Judgment affirms the co-existence of two or more distinct attributes in the same objects. This is a distinction which turns on facts, not on words, and cannot be obliterated by the growth or decay of languages: and the most perfect language (at least for Logical purposes) is that which brings out the distinction most clearly, and places it in the strongest light.

The theory of the late Dean Mansel on this subject is peculiar and not easily intelligible. When the Synthetical Judgment is once formed, he thinks, the mind proceeds to make up a whole composed of the predi-

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it comes to include the whole connotation of the predicate. Of course there is a further element of confusion when the connotation of either term is in the transition state.

cate and subject, and the Judgment in question can afterwards be arrived at by merely analyzing this whole into its component parts. This view differs from the former by not making the subject become the whole, of which the predicate becomes a part, but representing both as parts of the same whole which is present to the mind, though apparently not identified with either of the terms separately. It seems a sufficient answer to this theory, that the analysis of such a whole could give us no information as to the mode of co-existence of the parts—as to which part was to be the predicate, and which the subject, or as to whether the predicate was to be quantified with *All* or *Some*. After I have ascertained that All gold is heavy, says Dean Mansel, that which was before conceived as gold is now conceived as heavy-gold. But does this new conception of heavy-gold become the subject or the predicate of the proposition? If the subject, the proposition *All gold is heavy* now means no more than that *All heavy-gold is heavy*, which would be true even if there was no heavy-gold in existence; but if the new notion, heavy-gold, becomes the predicate, the meaning is *All gold is heavy-gold*, which is exactly the same assertion as our former one that *All gold is heavy*. Some Logicians indeed have laid it down that an adjective cannot form the predicate any more than the subject of a proposition; and that to put the proposition into Logical form, the word Object or Thing must be added. The proposition now before us should, according to them, be expressed for Logical purposes, All gold is a heavy

object or thing. It might be more correct to add the name of the subject than the indefinite *Object* or *Thing* if any substantive is required; and in those languages where the adjective has different forms for different genders (such as the Latin), we find that the predicate is usually made to agree in gender with the subject, and not with Object or Thing supposed to be understood. *All gold is heavy-gold* seems to be a better expression for what usually passes in the mind than *All gold is a heavy thing*, although the latter mode of expression may be more convenient for the purposes of Logic. This being borne in mind, the doctrine that the predicate is the whole of which the subject is a part is shown to be consistent with the existence of Synthetical Propositions; but if the subject is represented as the whole, and the predicate the part, all propositions become Analytical. Dean Mansel's expressions render it difficult to ascertain which was meant, or rather he does not seem to have made up his own mind on the matter.

CHAPTER XIV.

PROPOSITIONS AND JUDGMENTS, AND THEIR FORMS.

THE subject of Judgments and Propositions has been, to a certain extent, anticipated in treating of Terms and Things, because the divisions of Terms and Things adopted by Logicians frequently involved a reference to them; and the same subject has been further touched on in the last Chapter. Thus what Mr. Mill calls the

Import of Propositions has been already discussed. As it is disputed whether terms stand for, or are names of (as it is often expressed), Ideas or Things, so it is disputed whether Propositions or Judgments express relations between Ideas or relations between Things: and we have here to deal with a third theory, namely, that they express relations between Words or Names. This last view may be briefly disposed of. For what is a Word or a Name? Not a mere sound, but a sound standing for either an Idea or a Thing; or rather for both, as already explained, since it usually has both a connotation and a denotation. Relations between Words or Names are not, therefore, mere relations between sounds, but relations between the Ideas or Things which these sounds stand for—relations between their connotations, their denotations, or both. Indeed the desire which some writers exhibit to stop at the name, instead of going on to consider its meaning, is to me very surprising. It is like gazing at the sign-board of a hotel or tavern, instead of going in to order what one requires. Possibly the theory in question was suggested by the fact that in Logic we may employ the letters of the alphabet in our exposition, and treat of no other kind of propositions than Every B is C. But Every B is C is meaningless, unless B is supposed to stand for a class of Things, and C is supposed to imply one or more attributes which are asserted to belong to all the members of of this class. We do not, therefore, get rid of the reference to Ideas and Things by adopting these symbols. We only put that reference into its most general form.

And we get rid of Names to the very same extent that we get rid of Ideas and Things; for B and C may stand for or represent any names whatever, and have no special reference to any particular name or class of names. The Import of Propositions, however, is easily determined. B is C, asserts the identity of the extension or denotation of the term B with that of the term C; while B is not C asserts that these two extensions or denotations are not identical. The proposition Every B is C, asserts that the whole of the extension of B coincides with that of C, but it gives us no information as to whether the extension of C exceeds that of B, or is identical with it. Some B is C, asserts that a part of the extension of B coincides with that of C, but whether with the whole of the latter extension or with a part of it only, is likewise left undecided. B is C, where B is a singular term, asserts that the individual B (which constitutes the whole extension of the term B) is included in the extension of the term C, if C is a general term, or identical with it if C is a singular term also. Such are the relations between the extensions or denotations of the terms, or the relations between Things (as I have hitherto employed the phrase), which are expressed by affirmative propositions, and it is easy to extend the same explanation to negatives. But all terms, except proper names, have a connotation or comprehension, as well as an extension or denotation, and propositions express relations between these likewise. In this way Every B is C, means that the comprehension of the term B is invariably found in the same objects with that of the term C, but it leaves

it undetermined whether the comprehension of C is likewise never found apart from that of B. Some B is C, affirms that the comprehensions of the two terms are sometimes found coexisting in the same objects. B is C, if the singular term B has a comprehension (which some singular terms have : for instance, The first Emperor of Rome), means that this comprehension is invariably found along with that of C; but if B is a proper name which has no comprehension (or at least no fixed and definite comprehension), the proposition still asserts that the individual object B possesses the attribute or attributes connoted by the term C; of which such a proposition as Socrates was wise affords an instance. In this way, then, all propositions (or at least all propositions except singular propositions) express relations between the connotations or comprehensions of the subject and predicate, which may be called relations between Ideas. But there is a third way of regarding Propositions or Judgments, namely, as expressing relations between Ideas *and* Things, or between comprehensions *and* extensions. This, we have just seen, is the best mode of interpreting a singular proposition which has a general term for its predicate (such as Socrates was wise), but it is equally admissible in the case of other propositions. Thus, All men are mortal, asserts that the comprehension or connotation of the term mortal — the attribute mortality—belongs to the whole extension or denotation of the term Man; and All B is C, asserts that the attribute or attributes connoted by the term C are found in

all the objects denoted by the term B. But on examination it will be found that these three ways of stating the Import of a Proposition are not independent of each other; and the question then arises, which is the fundamental one—that which expresses the Import of the Proposition most fully, and from which the others may be derived? And here the principles already laid down are sufficient to enable us to arrive at a decision, at least where the predicate and subject of the proposition are general terms. For we have seen that it is the connotation which determines the denotation—the comprehension which determines the extension. The denotation of the term Man is simply all the objects in which the attributes connoted by that term are to be found; and the only way to determine whether the object before us is included in the denotation or not is to compare it with the connotation, and ascertain whether all the attributes comprised in the connotation are to be found in the object. The fullest and best statement of the Import of a Proposition, then, is to be found in the relation between the connotations of the subject and predicate which it asserts. And that the denotation is not the fundamental element (at least in the case of the predicate) is evident from the form in which propositions are usually expressed: for the predicate is most frequently an adjective, which connotes a distinct attribute, but whose extension or denotation cannot be ascertained until we determine what substantive is to be understood in connexion with it. Thus, while the connota-

tion of the term wise is well known, the denotation or extension of the predicate of the proposition Some men are wise, will be different according as we consider the adjective *wise* as the equivalent of wise beings or wise men. Logicians have generally adopted the former interpretation, while the structure of Languages, as already remarked, favours the latter. But the mere facts, that either substantive may be supplied, and that the denotation of the predicate will differ with the one which is supplied, proves that the connotation of the predicate is the only thing which really requires to be attended to. It is, moreover, certain that when we employ an adjective as a predicate, we do not invariably think of the objects in which the attributes connoted by it are found as constituting a class. I may judge that All men are mortal, without framing in my mind a class of mortal beings. I may, as Mr. Mill remarks, be thinking of nothing but man and the attribute mortality as belonging to *him*. In this case it would appear that the best way of expressing the mental process in question would be, All men are mortal *men*. But since all mortal-men are included in the wider class mortal-beings, if it is true that All men are mortal-men, it is equally true that All men are mortal-beings. The former proposition includes the latter, or rather, indeed, both propositions assert the very same matter of fact; and, therefore, in throwing propositions into Logical form we can adopt the rendering which answers our purpose best: and the purposes of Logic

seem to be best served by supplying the indefinite Object or Thing.*

In speaking of Judgments and Propositions as corresponding with each other exactly, it is necessary to call attention to a difference between the ordinary use of the two phrases, which might otherwise lead to confusion. A proposition is regarded as equally a proposition whether I believe it or not, and even if I disbelieve it: but when it is said that I *judge* or form a *judgment* that B is C, it is understood that I believe it. And hence many writers have introduced discussions on the nature of belief into their treatises on Logic, and disputes have arisen as to whether judging consists in merely comparing (or putting together and separating) ideas, or whether it includes somewhat more. But in a treatise on Logic we require words to describe the mental processes which form the equivalents of a Term and a Proposition. The word in our language which seems best adapted for the latter purpose is Judgment, and it is likewise that usually adopted by Logicians. And as the word Proposition

* For the rules already laid down as to the quantity of the predicates of affirmative judgments or propositions, suppose that Object or Thing is understood where the predicate is an adjective. If the name of the Subject were understood, the predicate of an affirmative proposition (when in the adjective form) would always be universal. If All men are mortal-men, All mortal-men are men: if Some men are wise-men, All wise-men are men. In order, therefore, to make our rules uniform, the ends of Logic are best served by understanding Object or Thing in connexion with an adjective Predicate.

does not imply belief (either in the mind of the speaker or of the hearer), if the word Judgment is to be used as its equivalent, we must equally divest it of the reference to belief which is involved in its popular use. Accordingly I have employed the term Judgment for the mental process which answers to a Proposition. Like the Proposition, the Judgment in this sense may be believed, doubted, or disbelieved by the person to whose mind it is present. If we employ the word Idea as the equivalent of Term, and the phrases "put together" and "separate" as the equivalents of the affirmative and negative copulas respectively, we may undoubtedly describe a Judgment as "putting together or separating two Ideas"; but this description is not likely to make it more intelligible to the reader. And although belief is not necessary to a Judgment as I use the word, neither does the mere putting together or separating of two Ideas constitute a Judgment. They must be so put together or separated as to become an object of belief, doubt, or disbelief. But how this differs from other kinds of putting together—for example, from the way in which the ideas of gold and of a mountain are put together in forming the idea of a golden mountain—will be more easily understood by reflecting on the process as it occurs in the mind, than by attempting to distinguish them in words. It is no easy task to explain by means of propositions the process by which propositions (or their mental equivalents) were originally formed.

It is necessary in Logic to provide a collection of Forms

to which *all* propositions can be reduced, because there is no proposition which may not be employed in a train of reasoning. But in these forms we should recognise *simple* propositions only, leaving composite or compound ones to be broken up into the various simple ones into which they may be analyzed. If any given proposition can be reduced to two distinct assertions, we should analyze it into these two. If both of these are used in the subsequent reasoning, we should analyze that reasoning so as to make each of them appear as a separate link in it; while, if one only is so employed, we may substitute that one for the complex proposition.* This is indeed the only practicable way of proceeding. If we allowed reasoners to combine as many distinct statements as they chose in a single proposition, and attempted to lay down rules for determining whether the conclusion was correctly drawn in arguments containing such propositions (as single steps), our task would be one of extreme complication, if not wholly impracticable. These observations must guide us in determining

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* There are, however, instances in which a complex proposition may be logically treated as simple. Thus the proposition All B and all C are D, can plainly be resolved into All B is D and All C is D; but we may, in many reasonings into which it enters, treat it as a simple proposition, having for its subject neither B nor C, but the wider class B-and-C. So, again, in the proposition All B is C and D, we may treat C-and-D as a single predicate (as indeed predicates, even when expressed by one word, often include several attributes). Such composite terms may be usually regarded as what I have called many-worded names; but occasions sometimes arise in which it becomes necessary to resolve them into their elements.

whether the forms of propositions recognised by ordinary Logicians, and adopted in the present treatise, are sufficient for all the purposes of Logic, or whether others must be added. These forms which we have recognised are the Categorical, the Hypothetical, and the Disjunctive, while the forms of Categorical propositions again are four in number, represented by the letters A, E, I, and O. On this list, it will be recollected, our table of valid and invalid Syllogisms was based. Are there, then, any *simple* propositions which cannot be reduced to these forms? Before answering this question it may be desirable to obviate an objection of a different kind. Hypothetical and Disjunctive propositions may appear at first sight to be composite propositions made up of two or more Categoricals. But it will be found on examination that this is not the case. Their subjects and predicates (if these terms may be used to denote their members) indeed usually are Categorical propositions, but then the Hypothetical or Disjunctive proposition includes not merely the subject and predicate, but also the copula, or expressed relation between them. This copula in the Hypothetical proposition is expressed by the particle "If," or the words "If—then", while in the Disjunctive it is "Either—or". And the consideration of this copula also enables us to justify our division of all propositions into Categorical, Hypothetical, and Disjunctive. A proposition, for instance, of the form If B is C either D is F or G is H, presents no difficulty. That it is a Hypothetical not a Disjunctive judgment is shown by the copula "If." It is not essential either to the Hypo-

thetical or to the Disjunctive proposition that its parts should be *Categorical* judgments. They may equally be Hypotheticals or Disjunctives. Wherever there are two propositions *of whatsoever kind* connected by the "If—then" we have a Hypothetical proposition. Wherever there are two or more propositions* *of whatsoever kind* connected by the "Either—or," there is a Disjunctive proposition. I have indicated indeed a mode in which Hypotheticals and Disjunctives may be reduced to each other, and both to Categoricals, but in this mode of reduction we do not reduce a single Hypothetical or Disjunctive proposition to two or more Categorical propositions. Neither Hypotheticals nor Disjunctives, therefore, are really composite, unless the Disjunctive proposition is understood to include an assertion that its members are exclusive of each other, in which case it becomes a real composite. For, if the proposition Either B is C or D is F, asserts not only that one of the Categorical propositions B is C and D is F is true, but

* It might appear as if the parts of a Disjunctive proposition were sometimes terms instead of propositions, but on closer examination this will be found not to be the case. Thus the full logical way of expressing the proposition Either B or C is D, would be Either B is D or C is D. *Either B or C*, is not a proposition at all. It asserts nothing; but when the words *is D* are added, it becomes two alternative assertions, both of which can be expressed as propositions, but not as terms. On the other hand, Every B is either C or D (or Every B is C or D), is in fact a Categorical proposition, the predicate being C-or-D. Its copula is the simple *is;* and probably the way of expressing it would be Every B-which-is-not-C is D. It is to this kind of proposition, however, that some writers would confine the term Disjunctive, which, like many other Logical terms, is somewhat unsettled in its meaning.

also that both cannot be true — in other words that one is false — we have two perfectly distinct statements, either of which is capable of forming a complete link in a train of reasoning without requiring any aid from the other. Thrown into the Categorical form, the one asserts that All cases-of-B-not-being-C are cases-of-D-being-F, while the other affirms that No case-of-B-being-C is a case-of-D-being-F. These are two distinct Categorical propositions, and it is impossible to derive or infer the one from the other. If, therefore, the Disjunctive proposition really includes both of them, we must exclude it from our list of simple propositions, while it would also be desirable to invent some new propositional form in order to obtain a more convenient mode of expressing the statement which is conveyed by the extremely awkward Categorical proposition All-cases-of-B-not-being-C are cases-of-D-being-F.

However, it is with regard to Categorical propositions that the question is chiefly raised, and the best way of stating the objections to our list of Forms is to state the additions put forward by Sir William Hamilton, as necessary in order to render it complete. Sir William Hamilton expands the four recognised Forms into eight, which is done by expressing the quantity of the predicate, and substituting for the rule that the predicate of an affirmative proposition is particular and that of a negative universal, the assertion that the quantity of the predicate in both instances is what it is expressed to be. His eight Forms are (1) All B is *all* C, which he designates by the letter U; (2) All B is *some* C, which he

designates by the letter A (inasmuch as Logicians had laid down the rule that in their proposition A, the predicate, though its quantity is unexpressed, is really particular); (3) Some B is *all* C, which he designates by the letter Y; (4) Some B is *some* C, represented by the letter I; (5) No B is *any* C, represented by the letter E (Logicians having laid down that in their E the quantity of the Predicate, though unexpressed, is really universal); (6) No B is *some* C, represented by the Greek letter η; (7) Some B is not *any* C, represented by the letter O; and (8) Some B is not *some* C, designated by the Greek letter ω. When the quantity of the predicate is thus expressed, Sir William Hamilton remarks that all Categorical propositions take the form of equations or inequalities. Thus the proposition All B is all C may be written All B = All C, while All B is some C may be written All B = Some C. The adoption of these new forms makes a very material change in the exposition of Logic. Conversion in particular is greatly simplified. Converting a proposition is merely writing the same equation in a different way. Thus All B = Some C is plainly the same equation with Some C = All B.

If it be objected that the Propositions U, Y, η and ω do not occur in practice, it would probably be answered that we meet in practice with assertions which may be reduced to these forms and cannot be adequately expressed by the ordinary A, E, I, and O. For instance, we find it asserted that B and C are identical, that C is a definition of B, that the classes B and C are coextensive, and

so on, all of which propositions should be logically expressed by the proposition U, and not by A; while Archbishop Thomson adds that Disjunctive propositions are always reducible to U.* If, therefore, we seek to refute the new theory, we must show either that Hamilton's U, Y, η, ω are reducible to the forms A, E, I, and O, or else that they are complex propositions, and that the elements of which they consist are reducible to the forms A, E, I, and O.

The first branch of this reduction seems easy enough in the cases of Y and η. It has been already remarked that the predicate and subject of a proposition are not always determined by the order in which the terms are placed; as, for example, in the proposition Sweet are the uses of adversity, Sweet is the predicate and Uses-of-adversity is the subject. Now as regards Y and η, it may, I think, be fairly contended that they are merely A and O, with the predicate written before the subject. Hamilton admits that A can be converted into Y, and that Y can be reconverted into A, while a similar relation exists between O and η (O being no longer inconvertible on this system); and when we have reduced propositions to equations or identifications, it seems strange to regard All B = Some C as a different equation (or a different form of equation) from Some C = All B. In both propositions we equate or identify

* This observation would only seem to be correct if the Disjunctive proposition implies that its members are exclusive of each other; in which case, as already remarked, it becomes a complex or composite proposition.

the very same things, nor can I see that even the order of thought is different. If I say Some B is all C, I am not thinking of the connotation (and we have seen that the connotation forms the chief element in the Import of Propositions) of the term C as found in some Bs and in nothing else, but of the connotation of the term B as found in all the Cs. Consequently C, and not B, is the principal object of thought and the true subject of the proposition. Again, if I say No B is some C, I cannot be thinking of the Bs either as possessing or as not possessing the attributes connoted by the term C, for the proposition gives me no information as to whether all or any of the Bs in fact possess these attributes. I can plainly be thinking only of some of the Cs as not possessing the attributes connoted by the term B, and therefore the proposition which has been called η is really O with the predicate written before the subject (and, I may add, its meaning thereby obscured). But even if the order of thought is different, still it is impossible to adduce any proposition in the form Y or η for which we cannot supply a precise equivalent of the form A or O (as those who reduce propositions to equations are bound to admit), and there is, therefore, no reason why we should encumber ourselves with two new and very awkward Propositional Forms, neither of which most probably will ever occur in practice, when every statement of fact which they could possibly convey can be expressed with equal fulness by the universally recognised forms A and O. When a Logician discovers any statement comprised in the pro-

position Some B is all C, which is not comprised in the proposition All C is some B, it will be time enough to admit Y and A as two distinct Forms of Propositions in Logic; but if this should ever be done, the mutual conversion of A and Y into each other must be given up. If after converting Y into A, we can get back to the original Y by the reconversion of A and *vice versâ*, it is manifest that neither of these propositions asserts more nor less than the other; and of the two Forms, the Logician will naturally adopt that in common use, and will require the other (if it should ever be met with) to be reduced to it when the reasoning is being thrown into Syllogisms. Very similar observations will apply to η and O.

The case is different, however, with regard to U and ω, neither of which is reducible to any *single* Proposition of the form A, E, I, or O. It only remains to inquire whether they can be reduced to two or more propositions of these forms. But before determining this, it is desirable to make some further remarks on the quantity of propositions, and the sense in which we are to understand the *All* and *Some* of Logic. Quantity in its ordinary signification is made up of a number of things *taken together*. This is not the case with the (so-called) quantity of propositions. As already observed, a proposition does not assert that the things denoted by the subject possess the attributes connoted by the predicate *when taken together*, but that each of them individually does so. All men are mortal, means that *each* man is mortal. It can hardly be said that all men collectively

are mortal; for, though each man in his turn will die, it has never been proved that the human race as a whole is destined to extinction — that a time will ever arrive when it could be said with truth All men have died. So Some men are black, means that there are some men *each* of whom is black. Ordinary propositions are thus, as has been noticed, distinguished from Algebraic equations. Equations assert relations between the two sides of the equation taken as wholes; but $mx = ny$ does not imply that each x is a y. On the contrary if m and n stand for different numbers, no individual x can be identical with any individual y. Algebraic terms are thus taken collectively, Logical terms distributively. Nor have propositions anything to do with equality further than that they assert or deny the identity of certain individuals comprised in the denotation of the subject with others comprised in the denotation of the predicate. Two classes may be perfectly equal as regards the number of objects comprised in them and yet there may not be a single individual common to the two, while again they may be very unequal in their extensions and yet have a number of members in common. This distinction between propositions and equations is not brought out clearly in the usual mode of expressing the forms of affirmative propositions, because the terms *All* and *Some* are capable of being understood collectively as well as distributively; but when we pass to the negative forms it becomes manifest. No B is *any* C is Hamilton's expression for the Universal Negative E, and *any* is plainly a distributive, not a collective term. Here, too, it be-

comes evident that a negative proposition cannot be described as the expression of an inequality. The proposition No B is C leaves it undecided whether B is a larger or a smaller class than C, or whether the two are of equal magnitude. It asserts that there is not a single object common to the two classes, but says nothing as to their relative numbers. We have further observed that though a proposition can always be *reduced* to a form in which the predicate stands for a class of things, it is not necessary for the thinker or reasoner to refer it to a class at all. When he asserts that All men are mortal, he may not be thinking of the class mortals, but only of the class men and the attribute mortality as belonging to each of its members: in which case he cannot be considering whether men constitute the whole of the class mortals or a part of it only. It can hardly be said that such a predicate has any quantity: and when Logicians state that its quantity is particular, the real meaning is, that what has been asserted justifies us, when converting the proposition, in quantifying it particularly but not universally; for if All men are mortal, Some mortals must be men, but All mortals need not be so. I now pass to another point. In the rules formerly laid down, it has been assumed that *Some* does not exclude *All*—that it is not to be treated as meaning Some *only*. Take for example the rules for Subalternation and Opposition. From the proposition All B is C we inferred that Some B is C, a proposition which would be inconsistent with the former if we understood Some to mean Some only. Hamilton, however, and some other

writers, seem disposed to depart from this practice and to employ the term Some in the sense of Some only. Either, it is contended, the predicate applies to the whole subject or it applies to a part of it only or it does not apply to it at all; and as the latter alternative is expressed by a negative proposition, the two Affirmative Forms recognised by Logic should be that which asserts that the predicate applies to the whole subject, and that which asserts that it applies to a part of the subject only. This would be plausible enough if men knew everything; but unfortunately, in the present state of our knowledge, we frequently know that some predicate applies to a part of a class, when we are unable to ascertain whether it is true of the remainder or not. For example, I have seen a large number of swans, and have conversed with persons who saw others, and I find that all the swans I ever saw or heard of were white. But then I know that there are great numbers of swans which I never saw or heard of, and I can see no reason why all swans should be white. How am I to put the knowledge which I now possess into the shape of a proposition? If I say All swans are white, I am stating more than I know to be true (and what in fact turns out to be untrue). If I say Some swans are white (*Some* being understood to mean Some *only*), I am likewise stating more than I know and more than I have any reason for believing; for in affirming that whiteness is an attribute of some swans *only*, I affirm that it is *not* a property of others. If, therefore, *Some* is understood to mean Some *only*, neither A nor I

(nor, I may add, U or Y) will express what I have ascertained; yet surely, the statement which I wish to make is a simple assertion not resolvable into two or more propositions, and I cannot see by what two or more propositions a Logician who understands Some in the sense of Some only would endeavour to express it.

Now, suppose I meet with a credible man who has been in Australia and who informs me that he has seen black swans there, and I desire to convey what I now know to others; is it not equally clear that what I now desire to state is not one thing but two—namely, that there are swans which are white, and that there are other swans which are not white? My former experience and information all led to the same result, but this last piece of information leads to a new one which I did not know or suspect before, and I have therefore two results to communicate instead of one. Yet if Some means Some *only*, these two results—these two statements—are expressed by the single proposition Some swans are white. But the same two results would on this theory be equally expressed by the proposition Some swans are not white: for if it is *only* Some swans that *are* white, there must be others which are *not* white, and *vice versâ*.* The two propositions I and

* Accordingly Dr. Murray enumerates as Compound Propositions, Exclusives, Exceptives, Comparatives, Inceptives, and Desitives—a statement which seems substantially correct even if his classification and analysis are not perfect. His example of an Exclusive proposition is, All except the wise man are mad. This is equivalent to the two propositions All non-wise men are mad, and The wise man is not mad. As an Exceptive he gives, Virtue alone is nobility. This is in fact an Ex-

O would thus become identical in meaning, and the Logician ought to recognise one of them only, laying the other aside as merely a different mode of making the same statement. Moreover, if we adopted this meaning of *Some*, it would not be possible to adhere to it consistently in our reasonings. Take for instance the Syllogism

All B is (some) C;
All B is (some) D;
∴ Some D is C.

Here, though it is *only* some Cs and some Ds whose identity with each other has been proved by showing that each of them are identical with the whole of the Bs, there is nothing to prevent the *other* Ds from being identical with the *other* Cs; nor on the other hand, have we any reason for believing that they are so. The premisses, therefore, do not warrant our conclusion that Some D is C in the sense of Some Ds *only* are Cs, while they do not enable us to infer that All Ds are Cs; and if we are to recognise no signs of quantity in Logic but *All* and *Some only*, I cannot see how we can draw any conclusion in this instance, though there is plainly a conclusion to be drawn.

But it is when taken in connexion with the Quantification of the Predicate, that the use of *Some* in the

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clusive otherwise expressed, and is resolvable into Virtue is nobility, and No non-virtue is nobility. The other kinds I need not enlarge on, but it seems to me that a Comparative proposition is erroneously classed as compound. If I say John is taller than Thomas, I do not imply that either John or Thomas is tall. I make the simple assertion that whatever the height of Thomas may be, that of John exceeds it.

sense of *Some only* attains its maximum of inconvenience. The proposition All men are mortal, does not justify us in asserting either that All men are *all* mortals, or that All men are some mortals *only*. But Hamilton would require every reasoner to quantify his predicates (if asked to do), and, apparently, only provides him with the two words *All* and *Some* (in the sense of *Some only*) for the purpose. Suppose, then, you make the assertion that Every misgoverned country is distressed, and I desire to combat this statement, both disputants adopting the Hamiltonian Theory of Logic. I commence by requiring you to quantify the predicate, and you quantify it with the word *All*. I immediately reply that misgoverned countries do not constitute all distressed countries, because there are many other causes of national distress; and if I succeed in establishing my point, you are defeated in the argument without the real import of your assertion having been even touched on. Again, suppose you quantify it with the word *Some* meaning *Some only*. I immediately call on you to prove the *only*—to prove that there are some distressed countries which are *not* misgoverned. You bring forward instances. I reply that in each of these instances the country in question *was* misgoverned; and it will be found that we have actually changed sides—I contending that national distress has always been caused by misgovernment, while you continue bringing forward instance after instance to prove that in it, at least, no such connexion existed. Such,

it seems to me, would be the course which the argument would probably take if both reasoners were Hamiltonians; but if a reasoner who had not adopted that theory was called upon to quantify the predicate of his proposition Every misgoverned country is distressed, he would meet the demand by a direct refusal, on the ground that the quantity of the predicate was wholly irrelevant to the point in dispute. In short then, if in our Logical Forms we were to use the word *Some* in the sense of *Some only*, we should have no means of expressing many of the assertions which are in every-day use: we could not carry out the exclusion consistently without depriving ourselves of the means of expressing reasonings which are undoubtedly valid; we should be involved in endless difficulties if we insisted on quantifying the predicates of propositions, or even on converting them, because the (implied) quantity of the predicate is involved in every conversion; and we should, moreover, have to express as simple propositions the results of two opposite sets of observations, while we should have no simple Propositional Form to express the result of either set of observations separately. On the other hand, if we employ the term *Some* in the sense of Some *at least*—in which sense it does not exclude all—we avoid all these difficulties, and moreover find it easy to reduce to our Forms all assertions that are really limited to Some *only*, though we should have to express them (in conformity with the facts which they are intended to convey) not by a single proposition but by a combination of two propositions. Some Bs *only* are Cs, is the

exact equivalent of the *two* propositions Some Bs are Cs and Some Bs are not Cs. And as to *All*, it would be better to use the term *Every* which excludes the collective use, and likewise corresponds better to the terms *Omnis* and πᾶς, used by the Latin and Greek writers on the subject.

It was necessary to fix the Logical meanings of the words *All* and *Some* before further examining Hamilton's Propositional Forms. This being done, we now proceed to our task. We commence with the proposition U, or All B is *all* C. If this proposition means that All the Bs taken collectively are identical with All the Cs taken collectively, this is a total departure from the ordinary meaning of the *All* of Logic. Nor could we descend with safety from such an assertion about classes taken collectively, to similar assertions respecting the individuals comprised in these classes. We should in doing so most probably fall into a fallacy of Composition or Division. For instance that All Soldiers are All Armies is true, if the terms are taken collectively; but no individual soldier is an army. Can it then be said that the meaning of the assertion is *Every* B is *every* C? Such an assertion would not be true, even if B and C were synonymous, or identical, terms. Is it true that Every Man is Every Man? Clearly not. In order that Every Man should be Every Man, it would be necessary that *each* individual man should be identical not only with himself, but also with every *other* man. Such an assertion could only be true if each of the two classes B and C

consisted of a single member. But Hamilton does not intend to convey by the proposition U either that the two terms B and C are to be identified merely as wholes, or that each member of the one is identical with every member of the other. What he desires to assert appears to be that every member of either class is equally a member of the other. Now this assertion, which is very badly expressed by the form All B is all C, can be perfectly expressed by a combination of two propositions of the usually recognised forms, namely All B is C, and All C is B—assertions which are totally distinct from each other and may have been arrived at by very different trains of reasoning or observation. Euclid, for example, gives separate proofs of the propositions that Every equilateral triangle is equiangular, and that Every equiangular triangle is equilateral, one of which the student learns before the other.*

And though Y has been already struck out of the list of Propositional Forms on other grounds, it may be noted that if the proposition Some B is all C affirms

* Hamilton's answer to this reduction of his U to two propositions, both of the form A, consists in rendering these two propositions by All B is *some* C, and All C is *some* B, and then taking "Some" in the sense of Some *only;* whence it would follow that these two propositions could not both be true: for All B is some C implies that C is a larger class than B, while All C is some B implies the contrary. But the rejoinder to this is twofold. First, there is no necessity for quantifying the predicates (except when we are converting a proposition), and secondly, if we quantify them with the word Some, this word does not mean Some *only*. Both propositions, as thus quantified, may therefore be simultaneously true.

that each one of Some Bs is identical with every C, we are involved in the same difficulty as in the case of U; while if the terms Some and All are used only in a collective sense, we could not apply the proposition Some B is all C to individual cases. Y is indeed not resolvable into two distinct propositions, because Some B is C and All C is B are not independent statements (like All B is C and All C is B), the former being merely the converse of the latter. But then this latter proposition expresses (and in a better way) all that was intended to be conveyed by the proposition Y. The form η has likewise been sufficiently dealt with, and consequently the form ω alone remains to be considered. It is expressed as Some B is not some C. No such proposition is ever met with in practice, and that for a very obvious reason. The word *Some* is wholly inapplicable unless the term to which it is annexed is a general name denoting more than one object. But whenever the terms B and C are general names the proposition ω must be true. As Archbishop Thomson (who only adopts Hamilton's Affirmative Forms) observes, it is true that Some salt is not *some* salt, because the salt in this cellar is not the salt in that. This kind of negative proposition is thus true even when the subject and predicate are identical. Nor does it imply that some of the objects denoted by the term which constitutes both subject and predicate differ from others in their properties; for though all Postage Stamps, for example, were precisely alike in their properties, it would still continue to be true that the Postage Stamp

on this envelope is not the Postage Stamp on that. No single member of any class is identical with more than one member either of that or of any other class ; and this statement sums up all the information that could be conveyed to us by any number of propositions of the form Some B is not some C. But though Hamilton does not bring out the point clearly, he probably intended to convey something different from this by his proposition ω. He probably meant it to be the Contradictory of his proposition U, and as U is really a composite, consisting of two propositions both of the form A (the predicate of each being the subject of the other), its Contradictory ω must likewise include two distinct assertions, both of which will be of the form O. Some B is not some C, thus interpreted, is equivalent to the two propositions Some B is not C, and Some C is not B.* But in this case ω should be excluded from our list of Propositional Forms, being a compound proposition resolvable into two distinct ones; and it must be added that if All B is all C is a bad expression for what Hamilton intended to convey by it, Some B is not some C, is a still worse mode of expressing what it was intended to assert.

Hamilton's list of Propositional Forms seems to

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* But it ought to include these two assertions disjunctively, not conjunctively. In contradicting a composite assertion, we do not allege that the whole of it is false, but that some part of it is so. The true contradictory of the compound proposition All B is C and All C is B, is Either some B is not C or some C is not B. If this is what is meant by ω, it is not a complex proposition, but then it is reducible to one of the ordinary forms of Disjunctives.

have been based on the notion that a single proposition could be made to express not only the relation which the extension of the subject bears to the predicate, but the relation which the extension of the predicate bears to the subject: by doing which it would combine what is expressed by two propositions of the ordinary Logical Form, the predicate of each being the subject of the other. As each of these two propositions may be of any of the four forms A, E, I, O, it would seem at first sight as if sixteen not eight Propositional Forms would be necessary to express the whole number of their combinations. But this number will be reduced on examination. Pairs which involve contradictory assertions must be rejected, since we cannot affirm two incompatible relations in a single statement. Setting out the entire list and examining it from this point of view, we find that of our sixteen combinations, four, namely AE, EA, IE, and EI, are excluded for this reason: (AO and OA are not excluded, because the subject of one being the predicate of the other, the two propositions are not inconsistent). The combination AA gives us the form U. AI simply gives us the form A, since I follows from A by conversion and therefore adds nothing to it. In like manner EE, EO, and OE afford us the simple E, unless *Some* is taken to mean *Some only;* in which case the combinations EO and OE will be rejected as incompatible. The combination IA affords the form Y. II simply gives us I (either of the Is following from the other by conversion); but the remainder do not so easily fit into their

places. Assuming that OO gives us Hamilton's ω, we should still have the combinations AO, OA, IO, and OI to account for; while Hamilton's η does not appear to be the precise equivalent of any combination. If indeed *Some* is understood as meaning *Some only*, the combinations OI and IO will reduce to either O or I, which two propositions will on this supposition mutually imply the truth of each other: but I do not see how we are to arrive at η. If, however, this is the true genesis of the theory, it becomes clearer than ever that most of these Forms do not represent *simple* propositions, but combinations which it is the duty of the Logician to analyse into their elements.

CHAPTER XV.

DEFINITION AND DIVISION.

Logic, in the view that I have taken of it, is not more concerned about Definitions than about any other kind of propositions, and indeed does not even regard Definitions as the most important kind. But Treatises on Logic have so invariably dealt with the subject, that it may be desirable to give some exposition of it here.

Definitions were usually divided into Nominal and Real Definitions, or Definitions of Names and Definitions of Things. Whether the following explanation is that which these Logicians would have offered, I shall

not determine. I will merely say that it appears to me to be the only valid distinction that can be drawn. I have spoken of the subject and predicate of a proposition as terms; but, in reality, it will be seen that the true subject and predicate are rather the connotations or denotations of these terms than the terms themselves. We have already seen that when I assert that All men are mortal, my thoughts are not fixed on the mere words or sounds, but on the things denoted by the term man, and the attribute connoted by the term mortal. But occasionally the subject of a proposition is a *mere* term and no more; in which case (if the proposition is affirmative), the predicate usually declares either the connotation or the denotation of the term in question. A proposition of this kind may be called a Nominal Definition or a Definition of a Name. It occurs, for example, when some one, in speaking to us, uses a term the meaning of which we do not understand, and we ask him for an explanation; and it is also usual in books where some term with which the reader is not supposed to be familiar is introduced for the first time. It is of no consequence whether the explanation of this term—which is given by the predicate of the proposition in question*—is contained in one word or in several. It is only necessary that it should be understood. A good

* It should be noticed that the word "Definition" is sometimes used for the predicate of the proposition, instead of the proposition itself. We may say, apparently with equal propriety, that the Definition of a triangle is—"A triangle is a figure bounded by three lines," or that it is "A figure bounded by three lines."

Dictionary contains hosts of these Nominal definitions. The propositions in which they occur are usually unquantified, or quantified with the particle "a", the subject, though a general term, being in fact treated as singular. A literal translation also abounds with such definitions. Anthropos is Homo, is a Nominal Definition of the term Anthropos. Whether any particular word can be defined in this way or not, depends on the number of synonymous expressions (whether consisting of one word or several) which the language in which the definition is made contains, and the number of such definitions may be increased if we are allowed to borrow synonymous words or phrases from other languages. Proper names, however, cannot be defined in this way. Nor can such Definitions be made by particular or negative propositions. They set forth the meaning of a word, which cannot be done either by stating what it does not mean, or by stating any property which belongs only to *some* of the things which it denotes; for the meaning (connotation) of a general term must be found in *everything* that it stands for (or denotes). But every affirmative proposition, universal or singular, in which we can substitute the word "*means*" for the copula "*is*," without altering the sense,* may be regarded as a Nominal Definition of

* The affirmative copula of Logic does not imply existence, nor the negative copula non-existence. B *is* C, does not mean B *exists* as C: it only means that *If B exists*, it exists as C. So B is not C, does not mean either that B or C exists, or that they do not exist. It simply means that *If B exists*, it does not exist as C. But though B does not exist as C, it

the term which forms its subject—at least, if the *whole* meaning, and not merely a part of it, is set forth in the predicate. Thus, for the proposition A triangle *is* a figure bounded by three lines, we may substitute, A triangle *means* a figure bounded by three lines without altering the sense, and the proposition in question may be regarded as a Nominal Definition of the term triangle. It states the meaning—the whole meaning—of that term, or at least would do so to a person who did not know the meaning before. Such propositions, however, may either state the meaning which the term has in ordinary use, or the special meaning in which the author intends to use it.

A Real Definition or Definition of a Thing differs from a Nominal Definition in this respect, that it not merely states the meaning of the term, but analyses it into parts, and sets forth the component parts with more or less detail. Every Real Definition is thus a Nominal Definition, but a Nominal Definition need

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may exist as something else, and something else may exist as C. Thus the copula never gives us any information as to the real existence or non-existence of the things spoken of and we must obtain that information from the subject and the predicate. But, as a general rule, in Synthetical Propositions real existence is intended to be asserted. For we have seen that such propositions assert that the connotations of the subject and the predicate, though distinct from each other, co-exist in the same objects; and the co-existence of two things seems to imply the existence of both. In such propositions we cannot substitute "*means*" for "is," without altering the whole assertion; which is otherwise evident, since, if the connotation (or meaning) of two terms is distinct, one of them cannot *mean* the other.

not be a Real Definition. For in the first place, it is clear that a Real Definition is only possible where the meaning or connotation of the Term is composite and made up of parts into which it can be analysed. Thus we cannot give a Real Definition of the term Colour, because the connotation of that term is simple, and cannot be resolved into parts. Simple Ideas, as Locke says, cannot be (really) defined, though perhaps this observation is not true of some of the ideas which he regards as simple. But, of course, there is nothing to prevent our having a term synonymous with Colour, or a number of words which, when taken together, would form a synonym of it. There is, therefore, no reason why Colour cannot be defined nominally. Again, the general terms Thing, Object, Being, &c., cannot be really defined, because they seem to connote nothing except the single attribute or property of existence (or, rather, supposable existence), which cannot be analysed into parts. Logicians have accordingly laid down that a *Summum Genus* cannot be defined—that is, *really* defined, for there may evidently be a Nominal Definition of it. Further, when the connotation of a word can be resolved into parts, a Nominal Definition need not resolve it into these parts. It may define the meaning, by means of a single synonymous word, as in our instance of Anthropos and Homo; but a Real Definition is necessarily an analysis.

The best Real Definition then would appear to be that which gives the most complete analysis of the connotation of the term defined—that which resolves it

into the simplest parts, and enumerates all these parts separately. But such a Definition would often be very troublesome, for the parts are sometimes very numerous, and we have no single words by which some of them could be properly described. Moreover, in many instances the meaning, or connotation, of a name as popularly employed is not precisely settled, and in such cases the only practicable course seems to be to give a complete enumeration of the parts that are settled, and to use some vague expressions to describe the rest. To avoid these inconveniences, Real Definitions do not always profess to analyse the connotation of the term defined into its simplest parts. They frequently seek only to analyse it into parts, some simple and some complex, or perhaps all of them complex, which, when taken together, include the whole meaning of the term, and each of which has got some special name appropriated to it in the language in which the Definition is framed. And, as already noticed, the Schoolmen, or followers of Aristotle, held that the best—indeed the only proper—mode of framing a Real Definition was to analyse the connotation of the Term defined into two parts, one of which was called the *Genus*, and the other the *Essential Difference,* both of which will usually be complex. This view was based on the assumption that there was only one proper mode of classifying Things, and that all things found their proper place in this universal classification. No term could be defined which did not stand for a class of things, and that class could not be the highest (since, as we have seen,

the *Summum Genus* is undefinable). The best way, then, of defining a class-name was by means of the class-name which stood immediately above it in this only proper or legitimate classification of things—the Proximate Genus, as it was called—and the difference between the connotations of the name defined and the name of its Proximate Genus. This difference, again, was held to be always reducible—or, at least, it was maintained that if we regulated the connotations of our class-names by the one proper classification of things, it *would* always be reducible—to some *one* thing, which was called the Essential Difference. Thus, for instance, the Scholastic Logicians defined Man as a Rational Animal—Animal being the Proximate Genus, and Rationality the Essential Difference. It would evidently be out of place in a Treatise on Logic to discuss at any great length the proper classification of things. That can only be determined by examining the properties of all things in the Universe, in order to consider how they should be classified. It is sufficient to say, that there is no one classification of things which is universally admitted to be the best—much less to be the only admissible one: nor, if this difficulty were surmounted, would it be true that the difference between the connotation of a class-name and that of the class which stands immediately above it in any known classification can always be reduced to a single thing. The example above referred to will suffice to show this. Why are we to refer Man to the class Animal as his Proximate Genus? Why not, rather, refer him to the class Biped, the class

Mammal, the class Vertebrate, or the class Warm-blooded Animal? Do not all these classes present important features of distinction, which should be recognised by Logicians, and would not any of them, conjoined with the term Rational, express the meaning of the term Man better than is done by the words Rational Animal? Again, Reason or Rational, as we have seen, is a very ambiguous term, and in the sense in which it distinguishes Man from the lower animals, there are good grounds for maintaining that it does not consist of any one mental faculty, but of several differences—some probably differences in kind, and others only in degree. Lastly, it seems clear that the word Rational does not express the whole difference between the connotation of the name Man and the connotation of the name Animal. If we had a conversation with Balaam's ass, we could hardly deny to him the term Rational, but we should not call him a Man so long as his external appearance remained unaltered. And, indeed, the external appearance of an infant affords us sufficient grounds for referring it to the class Man before it has exhibited any evidence of possessing Reason, and even though it should ultimately prove to be an idiot. The theory is thus on every ground unsustainable. There are, no doubt, some words whose connotations can be expressed by a Proximate Genus and an Essential Difference, and when this can be done the Definition by Proximate Genus and Essential Difference is a Real Definition of the word. But it is neither true that all definable words can be defined by

a Proximate Genus and Essential Difference, nor that the Proximate Genus and Essential Difference affords the only, or even the best, Definition of the terms that can be so defined.

But the word Definition is often used not so much in relation to the proposition itself as to the intention of the person who employs it. A writer may give us a complete analysis of the connotation of a term without asserting its completeness or putting it forward as a definition, while on the other hand he may give us an incomplete or erroneous analysis and yet assert its completeness, and thus put it forward as a definition. Now, the word Definition is sometimes employed for any proposition which *purports* to be a definition, even though the analysis which it sets forth proves to be wholly erroneous. It is in this sense that the word definition is used when we speak of Aristotle's definition, the Scholastic definition, or Mr. Mill's definition, of any particular term. We mean by definition in such cases the propositions which Aristotle, the Schoolmen, or Mr. Mill put forward as definitions. Thus I have stated that the Scholastic definition of Man was a Rational Animal, but I have also pointed out that the connotation of the term Man includes something more than the two parts Animality and Rationality; and, therefore, if by a Definition we mean a proposition which sets forth the whole connotation of the term which forms its subject (analysed into two or more elements), the proposition Man is a Rational Animal is not a definition at all. A definition thus means either a proposition in which

the predicate contains an analysis of the whole connotation of the subject, or a proposition in which this is *asserted* to be the case, although in fact it may not be so. We usually employ the term Definition in this latter meaning when we speak of a good or a bad definition. What is a bad definition in this meaning of the word is no definition at all in the former one. And while a definition in the former sense is a single proposition, a definition in the latter sense consists of two distinct propositions. Thus, in the case we have considered, one of these propositions is Every Man is a Rational Animal (which is true), and the other is This proposition is the definition (in the former meaning) of the term Man; which last assertion is not true, because, though the connotation of the term Man *includes* Rational Animal, it contains something more. In this meaning of the word Definition, moreover, any term may be defined: for I can *assert* that any term has a connotation, that this connotation consists of parts, and that the parts of which it consists are those comprised in my enumeration. Whenever I assert this, I define the term in question in the sense now explained.

The best Definition* of a term in popular use is that which sets forth the whole connotation of the subject without unnecessary repetition, analysed into the ele-

* It is erroneous to speak of *the* definition of any term, for there is hardly any term which does not admit of more than one definition. This mode of speaking has come down to us from the Schoolmen who maintained that there was but *one* way of (really) defining a term—namely by the Proximate Genus and Essential Difference.

ments which (whether simple or complex) are most convenient for ordinary purposes—that is, usually, into the elements which are best known and most familiar. What is the best scientific definition of a scientific term, however, is a different question. Here we are really forming a new term (for an old word with a new meaning constitutes a new term just as much as if a new term were introduced), and the real question to be decided is What should the connotation of this new term be in order that it should prove most serviceable in the Science we are dealing with? As soon as we have determined what this connotation is to be, the rules for defining such a term are the same as those for defining a term in common use; but the definition is considered a bad definition if the term as thus defined is one which is rarely employed in important propositions in the Science, or one which cannot enter as subject or predicate into important propositions without adding other words to qualify it. Such a term is in reality rather an ill-selected than an ill-defined term; but the usual way of amending its faults is not to introduce a new word, but to re-introduce the same word with a new definition—that is with a new connotation. This, as has been remarked, is really introducing a new term, and, at the same time, dropping an old term (alike in sound with the new one), which has proved unserviceable in practice. And where a term is an exclusively scientific one it is comparatively easy to drop it and to introduce a new one.

The phrases Real Definition and Definition of a

Thing (or rather of a class of Things, for an individual Thing can seldom, if ever, be *really* defined) have, however, been variously used, and according to Mr. Mill the distinction between a Real Definition and a Nominal Definition is different from what has just been stated. A Nominal Definition, according to him, merely states the meaning of a name, and in it we may substitute "means" for "is" without altering the import of the assertion; but a Real Definition assumes, in addition to the meaning of the name, the existence of a corresponding Thing or class of Things. I believe, however, that it will be found that no definition assumes the existence of a Thing or class of Things. What really happens is, that a definition often occurs in a work in which the existence of a corresponding class of Things is assumed or asserted, and perhaps a description of the Things in question as an existing class is given as an introduction to the definition. But in such cases the assumption or assertion of a corresponding class of Things is not made by the definition itself, but by the context in which it is found. If the definition contained such an assumption it would evidently be a Complex Proposition involving two assertions, viz., that the term B means C and that a class of Bs exists;*

* But does not a Real Definition at all events assume the existence of the *Idea* of a class of Things? Certainly; but so does any definition. A definition declares the connotation or comprehension of a term, and this connotation or comprehension is (in the sense in which I use the word) Idea. But it is not implied in any definition that this connotation or comprehension—this Idea—can be realised as a picture in the imagination, and many writers maintain that no *general* Idea can be thus realised.

propositions which the Logician should deal with separately.

Other writers, again, apply the term Real Definition to any proposition in which the extension or denotation of the predicate is identical (or is *asserted* to be identical) with that of the subject, no matter what relation may exist between their connotations or comprehensions. Thus Rational-Biped would be a Real Definition of Rational-Animal, because it so happens that all Animals possessed of Reason are Bipeds; and conversely Rational-Animal would be a Real Definition of Rational-Biped. So also, Two-handed would be a Real Definition of Risible and *vice versâ;* for so far as we know, all two-handed animals are capable of laughing, and all animals capable of laughing are two-handed. Equiangular Triangle, again, would be a Real Definition of Equilateral Triangle, while Equiangular Quadrilateral would not be a good Real Definition of Equilateral Quadrilateral. Logic has evidently no peculiar concern with propositions of this kind. If it is *asserted* that the two classes (the subject and predicate of the so-called Real Definition) are coextensive, the Logician resolves the entire statement into two propositions of the form Every B is C and Every C is B. If it is not asserted, the Logician has no way of distinguishing between a proposition which possesses this property and any other universal affirmative proposition. It may be remarked that writers who refuse the name of Definitions to such propositions often apply to them the term Descriptions; but, in fact, every Universal Affirmative

Proposition contains a Description of its subject—unless we are to limit the term Description to such a description as would enable us to distinguish with certainty the objects denoted by the subject from every other object.*

Along with Definition, Logicians have usually treated of Division. It is indeed generally conceded that Logic has nothing to do with dividing an individual object into parts, as, for example, with dividing Ireland into Ulster, Leinster, Munster, and Connaught; but it has been supposed that it is concerned with dividing a class of Things into sub-classes. Logic, however, can tell us nothing as to the number of sub-classes into which the class B should be divided, or how each of these sub-

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* Logicians have laid down rules for defining, which may be briefly adverted to. A Definition should set forth the *entire* comprehension or connotation of the subject or term defined, this being what it professes to do. It should not contain anything which is not comprised in this comprehension or connotation; otherwise it gives us not only what it professes to give, but something more—it gives us for the contents of a notion the real contents, together with something which is not contained. It should not enumerate the same attributes more than once, and it should carry the analysis of the connotation of the term defined as far as is requisite for the author's purpose—on which latter point no precise directions can be given. Another rule is, that it should not enumerate any attribute which is derivable from one or more of the others which have been enumerated. This rule, however, is really a caution not to put such derivative attributes into the connotations of our general terms. Otherwise, while the connotation of the term would be improved by dropping this part, it is evident that so long as it is not dropped it ought to appear in the Definition. This last fault is that which Archbishop Whately designates Tautology.

classes should be distinguished from the others. That must be determined by the special properties or attributes of the Things contained in the class. It is alleged indeed that Logic enables us to divide all the Bs into the Bs which are Cs and the Bs which are not Cs (or more briefly into Cs and not-Cs). But Logic does not supply us with the term C, and after we have obtained this term there are two cases in which the proposed division fails, namely, where all the Bs are Cs and where none of them are so. In either of these events the class B remains as whole and undivided as before; and whether they have occurred or not cannot be ascertained by Logic. This Division by Dichotomy, as it is called, is therefore as much outside the province of Logic as any other kind of division. Logicians have laid down rules for Division as well as for Definition, but these are in fact obvious to any one who understands the meaning of the word Division. All the sub-classes taken together must make up the whole class which is to be divided, and nothing more: for if they do not make up the whole class, our division does not include all the parts of the class divided, and if they make up more than the whole, it is not a division of the class in question, but of that class *and something more*, while some of the alleged sub-classes will not really be sub-classes at all. Another rule is, that the sub-classes must not overlap each other, or that no two or more of them should include the same individual or individuals, this being deemed a fault in classification. Thus, if I were to divide Irishmen into Protestants and Roman Catholics

(or more fully into Protestant Irishmen and Roman Catholic Irishmen), the division would be faulty, because there are some Irishmen who are *neither;* while, if I were to divide them into Roman Catholics and Conservatives, this division, in addition to the foregoing fault, would exhibit another, since there are some persons who are *both*. It is generally possible to interpolate intermediate classes between the class to be divided and the sub-classes which it is divided into: and in this case the rule has been laid down that the same number of intermediate classes should intervene between the principal class and each of the sub-classes enumerated. Thus, if I divided Undergraduate students into Sophisters, Senior Freshmen, and Junior Freshmen, the division would be considered objectionable, because there is no intermediate class between the principal class Undergraduates, and the sub-class Sophisters, while Freshmen is evidently an intermediate class between it and the sub-classes Senior Freshmen and Junior Freshmen. Such divisions, however, are rather awkward than erroneous. A class can usually be subdivided on several different principles. Thus we might subdivide the class Undergraduates with reference to their collegiate standing, as in the above instance, with reference to their religious belief, with reference to the professions for which they were studying, with reference to their residence or non-residence within the walls, with reference to their obtaining or not obtaining College Honors, or with reference to their moral conduct and amenity to discipline—and indeed on a number of other

principles also. Divisions conducted on different principles are called cross-divisions, and it will often be found that what is put forward as a division of a class contains sub-classes belonging to two different cross-divisions, and, as a natural consequence, that the same individuals are frequently found in two or more of these sub-classes.

Some Logicians have confined the term *Disjunctive* Proposition to a proposition of the form Every B is either C or D (or Every B is either C, D, F, or G), and they describe such a proposition as asserting that the class B is divisible into the sub-classes C and D (or the several sub-classes disjunctively enumerated by the predicate). Now as it is essential to a good division that the several sub-classes should not overlap each other (or contain any individuals in common), it is contended that such a Disjunctive Proposition implies that the various sub-classes enumerated by the predicate are exclusive of each other, and that we can therefore argue from the assertion of one to the denial of all the rest—a mode of reasoning which has been rejected in this treatise as invalid. But in the first place, if we are to confine the name Disjunctive Judgment to a proposition of the form Every B is either C or D, how are we to designate a proposition of the form Either B is C, or D is F? Such propositions are met with in practice, and our table of the Forms of Propositions ought to be wide enough to include them. But further, the particles "Either—or" do not necessarily imply the division of a class even when the pro-

position is of the form Every B is either C or D. As already noticed, if I allege that Virtue recommends us either to the approval of mankind or to the favour of the Deity, I do not imply that it may not have both effects. And in fact we frequently know that one *at least* of two assertions is true, when we are not in a position either to say that one *only* is so, or that *both* are so. We therefore require a form of words to express our knowledge when it is in this condition, and those who refuse to allow us the Disjunctive Proposition for this purpose are bound to give us something else in its place. On the other hand, *their* Disjunctive is easily explained as a composite, consisting of two distinct propositions, viz., Every B is *either* C or D, and No B is *both* C and D.

CHAPTER XVI.

OF INFERENCES AND SPECIALLY OF REDUCTIO AD IMPOSSIBILE.

Of immediate inferences—Subalternation, Conversion, and Obversion, together with Contradiction and Contrariety—enough, perhaps, has been already said. There are, however, immediate inferences of other kinds, for example, what has been called Immediate Inference by Added Determinants. Thus from the proposition A Negro is a Fellow-creature, we may infer A Distressed-

Negro is a Distressed Fellow-creature. In general, if All Bs are Cs it must be true that All Bs-which-are-Ds are Cs-which-are-Ds; and a similar inference can be drawn in the case of an Universal Negative. But in the case of a Particular proposition no such inference can be drawn. From the proposition Some Bs are Cs we cannot infer that Some Bs-which-are-Ds are Cs-which-are-Ds; for though Some of the Bs are Cs, all the Bs which are Ds may be found among the remainder of the Bs. It would be no easy task to draw out a complete list of these immediate inferences; and the only general rule would seem to be, that whenever the alleged Immediate Inference contains the same assertion, or a part of the same assertion, with the proposition from which it is inferred, the inference is valid; but otherwise not. We may illustrate this by a short examination of the Immediate Inferences which may be drawn from an Universal Affirmative. From the proposition All B is C, we may infer that wherever the attribute or the whole of the attributes connoted by the term B is present (whatever other attributes may be present along with them), the attribute, or any one or more of the attributes, connoted by the term C are likewise present (of course along with the attributes, or any portion of the attributes, which are present along with those connoted by B). We may likewise infer, that on *some* of the occasions when any portion of the attributes connoted by B are present, the whole or any given part of those connoted by C are present also; but here we must abstain from adding any additional attri-

butes to either subject or predicate (as now altered). If we can affirm any predicate—for example Biped—of All Rational Animals, this predicate or any part of it may be affirmed of Some Animals, because All Rational Animals are Some Animals; and for the same reason it may be affirmed of Some Rational Beings. But while we can infer from All Rational Animals are Bipeds, that Some Animals are Bipeds, we cannot infer that Some horned Animals are horned Bipeds; and in fact it so happens that among the (some) animals which are bipeds there are none which possess horns. If then we introduce new attributes into the subject, we must preserve *all* the attributes comprised in our previous subject in order to draw any inference, and we can introduce no new attributes into the predicate unless they are included among those which we have already introduced into the subject. When all the attributes included in the previous subject are preserved, we can introduce as many more as we please, and at the same time introduce these or any part of them into the predicate without affecting the truth of our assertion. We can likewise drop any part of the connotation of the predicate without affecting the truth of the proposition; but if we drop any part of the connotation of the subject we must cut down the Universal proposition into a Particular one. These are the rules for A. Similar rules may be laid down for E, I, and O, which I shall leave the reader to trace for himself.

Passing now to the Syllogism, it has been noticed

that Aristotle regarded his Dictum as the sole principle involved in the Categorical Syllogism, although the Dictum applies directly to Syllogisms in the First Figure only. The manner in which he sought to verify most of the valid forms of the other three Figures has likewise been explained; but the defect of the system seems to be that, while it proves the validity of the modes which Aristotle succeeds in reducing, it does not so readily establish the invalidity of the rest. It is no easy task to prove that no other modes than those which appear in Aristotle's list can be reduced to the First Figure by means of these processes; and even when that is accomplished, it does not follow that all other modes are invalid. The reduction described at p. 57 was called Ostensive Reduction, and Aristotle and his followers confessed that there were two valid modes—those known as Baroko and Bokardo—which could not be reduced in this manner.* Accordingly, a different method of establishing the validity of these modes was hit upon, which was termed *Reductio ad impossibile.* The Conclusion of a Syllogism must be admitted to follow from the Premisses, if the truth of both Premisses necessarily

* For in these two modes one of the Premisses is O, which cannot be a Premiss in the First Figure; and, since O is inconvertible, the difficulty cannot be removed by conversion. To accord with the Dictum, the Major Premiss must predicate something of a *whole* class (which is not true of O, since it is particular), and the Minor Premiss must assert that something is *contained in* that class (which is not true of O, inasmuch as it is negative). The same result would follow from the Special Rules of the First Figure already laid down, which may be deduced from the Dictum, as well as from our Axioms.

implies the truth of the Conclusion. But the truth of both Premisses implies the truth of the Conclusion whenever the falsity of the Conclusion necessarily implies the falsity of one (or both) of the Premisses; since in that case if both Premisses *had been* true, the Conclusion must likewise have been so. Now, in *Reductio ad impossibile* we seek to prove that *if* the Conclusion is false, one *at least* of the Premisses is false likewise; which being established, the validity of the Syllogism is proved. But the falsity of the Conclusion is the same thing as the truth of its Contradictory; and if, therefore, assuming that this Contradictory *and* one of the Premisses are true, we arrive, by a Syllogism in the First Figure, at a Conclusion which *conflicts with* the *other* Premiss, we shall have proved the validity of the mode under examination. The Premiss which we employ along with the Contradictory of the Conclusion in the new Syllogism is called the Retained Premiss, while that which the new Conclusion is required to contradict is called the Suppressed Premiss. When this new Conclusion is reached, we have established that *if* the original Conclusion is false (that is, if its Contradictory is true), either the Retained Premiss, or the Suppressed Premiss, or both of them, are false; whence it follows, again, that if both Premisses are true, the original Conclusion is true also.* (The reader should

* It has been objected to the Aristotelian theory, that the whole of this reasoning cannot be exhibited in a Syllogism, or series of Syllogisms, in the First Figure. The objection is not, I think, well founded, though

bear in mind that we have not proved that if the original Conclusion is false, the Suppressed Premiss is false. The falsity may, on that supposition, be in the Retained Premiss, and if so the Suppressed Premiss may be true. The falsity of either or of both of the Premisses answers our purpose equally well; but the Suppressed Premiss has been proved to be false, not on the single supposition that the Conclusion is so, but on the supposition that the Conclusion is false *and* the Retained Premiss true.) Such being the nature of *Reductio ad impossibile*, let us inquire when it will be possible, by substituting the Contradictory of the Conclusion* for one Premiss, to arrive by a Syllogism in the First Figure at a new Conclusion, whose truth is *inconsistent* with that of the other Premiss. The original Syllogism is called the Reducend: the new one is called the Reduct. Our first step here is to inquire what is the effect

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the whole of the reasoning cannot be expressed in a *single* Syllogism in the First Figure. Let us take, for example, the Mode Baroko. Here one Syllogism would be Every Syllogistic Mode, in which the Conclusion is true whenever both Premisses are true, is a legitimate Mode: Baroko is such a Mode: therefore Baroko is a legitimate Mode. But it is further requisite to prove that Baroko is a Syllogistic Mode, in which the Conclusion is true whenever both Premisses are so, and the proof of this would extend over more than one additional Syllogism, though, I believe, without requiring any other Figure than the First. Some of the reasoning might be best exhibited in Hypothetical Syllogisms, the mode of reducing which to Categoricals has been already explained.

* Questions are sometimes asked as to how the process would be affected if the Contrary, or Sub-contrary, of the original Conclusion was

of substituting the Contradictory of a Conclusion for one Premiss of a Syllogism, as regards the terms of the new Syllogism and their position. Since the Contradictory of any proposition has the same subject and predicate with that proposition itself, the effect in this respect will be the same as if the original Conclusion were substituted for a Premiss, and we may, therefore, inquire equally what would be the effect of this latter substitution. First, what will be the Middle Term of the Reduct? The answer is, the Extreme of the Reducend Syllogism which occurs in the Retained Premiss. For the only term which occurs in *both* Premisses of a Syllogism is the Middle Term of *that* Syllogism. But both extremes of the Reducend Syllogism occur in its Conclusion (and in the Contradictory of that Conclusion, whose terms are the same as those of the Conclusion). Hence the extreme of the Reducend which

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substituted for it, and if, from this substituted proposition and one Premiss, we deduced a new Conclusion, which was the Contrary, or Sub-contrary, of the other Premiss. Such questions are easily answered, provided we clearly understand the nature of the process we are employing. We must set out with a proposition *whose falsehood implies the truth of the old Conclusion*—which is the case with its Sub-contrary, but not with its Contrary—for our object is to prove that if the Suppressed Premiss (as well as the Retained) be true, this proposition *is* false, *and, therefore, the original Conclusion is true.* And we must arrive at a new Conclusion, *whose truth implies the falsehood of the Suppressed Premiss*—which is the case with the Contrary, but not with the Sub-contrary of that Premiss—for what we want to prove is, that if both Premisses of the new Syllogism are true, *the Suppressed Premiss must have been false.*

occurs in the Retained Premiss (*i.e.* the Major Term of the Reducend if the Major Premiss of the Reducend is retained, and the Minor Term of the Reducend if the Minor Term is retained) will become the Middle Term of the Reduct, since it will occur in both Premisses of the Reduct—in the Retained Premiss and in the old Conclusion (or its Contradictory). Having thus determined the Middle Term of the Reduct, we have next to ascertain what are its extremes. Here it must be premised that in *Reductio ad Impossibile,* as Logicians have hitherto employed it, the Retained Premiss always remains the *same* Premiss as before—if the Retained Premiss was the Major Premiss of the Reducend it is made the Major Premiss of the Reduct, and if it was the Minor Premiss of the Reducend it becomes the Minor Premiss of the Reduct.* But the terms of this Retained Premiss were the Middle Term and the proper Extreme of the Reducend. These have now become the Middle Term and the *same* Extreme of the

* It is easy to ascertain the consequences of changing the place of this Premiss on passing from the Reducend to the Reduct. Its Extreme has already been proved to be the new Middle: the old Middle (the other term in it) must, therefore, have become an Extreme. And it must (in the Reduct) be the extreme of the Premiss in which it occurs. But, on the supposition we are now making, the Premiss has *changed* its name, in passing from the Reducend to the Reduct. Hence the old Middle becomes, not the Extreme of the Retained Premiss, but the extreme of the Suppressed Premiss (*i.e.* it is the Major Term of the Reduct if the Major Premiss of the Reducend was suppressed, and the Minor Term of the Reduct if the Minor Premiss of the Reducend was suppressed), while the Extreme of the Suppressed Premiss changes its name, and becomes the *other* extreme (of the Reduct).

Reduct, since the Retained Premiss is the *same* Premiss of the new Syllogism as of the old. But the Extreme of the Reducend has become the Middle Term of the Reduct, and consequently the Middle Term of the Reducend must have become the proper Extreme of the Reduct—a result which may be briefly stated, *The Middle Term* (of the Reducend) *and the Extreme of the Retained Premiss have interchanged denominations* (on that Premiss becoming a Premiss of the *Reduct*). And a second rule follows, viz., *The Extreme of the Suppressed Premiss is the same Extreme in both Syllogisms*—for the third term of each of these Syllogisms is that which was the Extreme of the Suppressed Premiss in the Reducend. It occurs in the Reduct Syllogism only in the old Conclusion (or its Contradictory), which is now substituted for the Suppressed Premiss, and is the same Premiss in the Reduct that the Suppressed Premiss was in the Reducend (*i.e.* it is the Major Premiss of the Reduct if the Major Premiss of the Reducend was suppressed, and the Minor Premiss of the Reduct if the Minor Premiss of the Reducend was suppressed). Consequently, this extreme remains the same extreme that it was before. Indeed, as we have already proved that the Middle Term and the *other* extreme of the Reducend exchange denominations on passing to the Reduct, there is no position left for *this* extreme to occupy, except its former one.

These results will be made clearer by applying them to a symbolical example. The effect of the substitution in question being evidently independent of the *Mode* to

which it is applied (though not independent of the *Figure*, as will be seen), I shall take the mode EIO (which is common to all the figures), and show the effect of substituting A, the Contradictory of the Conclusion O, for each Premiss. (The Suppressed Premiss is written in brackets below each pair of Reduct Premisses, for convenience of comparison.)

Fig. 1.—	No M is P.	All S is P.	No M is P.
	Some S is M.	Some S is M.	All S is P.
	Some S is not P.	(No M is P).	(Some S is M).
Fig. 2.—	No P is M.	All S is P.	No P is M.
	Some S is M.	Some S is M.	All S is P.
	Some S is not P.	(No M is P).	(Some S is M).
Fig. 3.—	No M is P.	All S is P.	No M is P.
	Some M is S.	Some M is S.	All S is P.
	Some S is not P.	(No M is P).	(Some M is S).
Fig. 4.—	No P is M.	All S is P.	No P is M.
	Some M is S.	Some M is S.	All S is P.
	Some S is not P.	(No P is M).	(Some M is S).

The first column here contains the Mode EIO in each of the four figures, using letters of the alphabet for terms. The second column gives the pairs of Premisses afforded in each case by retaining the Minor Premiss, and substituting the Contradictory of the Conclusion for the Major Premiss, while the third column gives the pairs resulting from retaining the Major Premiss, and substituting the Contradictory of the Conclusion for the Minor. It will be seen, on looking at the pairs of Premisses in the second column, that

the original Minor Term has invariably become the Middle; M, the original Middle, has become the Minor; and P, the original Major Term, is the Major Term of the new Syllogism (or new pair of Premisses), as well as of the old. On the other hand, referring to the pairs of Premisses in the third column, where the Major Premiss of the Reducend has been retained, P, the original Major Term, has in all cases become the Middle of the Reduct, M, the original Middle, has become the Major Term of the Reduct, while S, the extreme of the Suppressed Premiss, is the Minor Term of both Reducend and Reduct. Another observation is suggested by this table. Our pairs of Premisses do not always form the Premisses of a Syllogism *in the First Figure;* and though in our examples they all warrant a Conclusion (and we might, therefore, have completed the Reduct Syllogisms), it has not been established that this will invariably be the case. The next step, therefore, is to ascertain what Premiss should be retained (or suppressed), in order that the Reduct Syllogism should be in the First Figure. This question is not difficult to answer. In the Second Figure the Middle Term (and, therefore, the Extreme) occupies the same position in the Minor Premiss that it does in the First Figure, while it occupies a different position in the Major Premiss. In the Third Figure this is reversed. The terms occupy the right position (meaning by *right* that which agrees, and by *wrong* that which disagrees with the First Figure) in the Major Premiss, but are wrong in the Minor. In the Fourth Figure the terms occupy

the wrong position in both Premisses. Reserving this latter Figure for subsequent examination, the rule applicable to the other two is—*Retain the Premiss in which the terms occupy a different position from the First Figure.* For we have seen that the Middle Term and the Extreme of the Retained Premiss interchange denominations when we pass from the Reducend Syllogism to the Reduct. If, therefore, the position was originally wrong, it is set right by the interchange. And not only will the order of terms in the Retained Premiss be correct, but it will also be correct in the new Premiss—the Contradictory of the old Conclusion. For it is to be observed that in the First Figure the extremes occupy the same place in the Conclusion that they have occupied in the Premisses. In that figure the Major Term is a predicate both in the Major Premiss and in the Conclusion, and the Minor Term is a subject both in the Minor Premiss and in the Conclusion: and it need hardly be added that if one term of a proposition be in the right place the other must be so. The terms of a Conclusion (or of its Contradictory), when substituted for a Premis, sare therefore always in their right places. If the Conclusion is substituted for the Major Premiss, the Major Term of the two Syllogisms is the same, and is in its right place in the substituted proposition; while if it is the Minor Premiss which is replaced, the Minor Term of both Syllogisms is the same, and that term is in its right place in the substituted proposition. Hence, too, it appears that in the Fourth Figure, whichever Premiss

we retain, we obtain a Syllogism in the First Figure; for, whichever Premiss we retain, the order of terms in the Retained Premiss was wrong originally, and is set right by the interchange, while in the Contradictory of the old Conclusion the order is always right.

If, then, in the Second Figure we retain the Major Premiss and suppress the Minor, or if in the Third Figure we retain the Minor Premiss and suppress the Major, the Reduct Syllogism will be in the First Figure, while in the Fourth, as already remarked, we may suppress either Premiss at pleasure. But we have still two further inquiries to make—first, when will the Reduct Syllogism be a *valid* Syllogism in the First Figure, *i.e.* when will it lead to *any* Conclusion? and secondly, when will the Conclusion so reached be *inconsistent* with the truth of the Suppressed Premiss?—that being what we are endeavouring to arrive at. Now, we have seen that the two Special Rules of the First Figure—that the Major must be Universal and the Minor Affirmative, result immediately from Aristotle's Dictum. If, then, we desire to apply *Reductio ad Impossibile* to a Syllogism in the Second, Third, or Fourth Figure, the Premisses of the Reduct must comply with these rules, in order that any Reduct Conclusion should be possible. But in the Second Figure the Major Premiss must be retained (in order that the Reduct should be in the First Figure), and hence it must be universal. Again, the Minor Premiss of the Reduct Syllogism must be affirmative. But it is the Contradictory of the Conclusion of the Reducend; and as two Contradictory propositions

differ in quality, the Conclusion of the Reducend must have been negative. Hence, if the Reducend be in the Second Figure, the Reduct will not be a *valid* Syllogism in the First Figure, unless the Major Premiss of the Reducend is universal, and its Conclusion negative. We thus arrive by a different path at the Special Rules of the Second Figure already laid down—that the Major Premiss must be universal, and the mode must be negative.

An examination of the Third Figure leads to a similar result. We must here retain the Minor Premiss; consequently, if the Reduct Syllogism (which is in the First Figure) is to be a valid one, this Retained Minor Premiss must be affirmative. Again, the Major Premiss of the Reduct must be universal. But it is the Contradictory of the Conclusion of the Reducend, and two Contradictories differ in quantity, as well as in quality. Hence the Conclusion of the Reducend must have been particular; and we thus reach the two Special Rules of the Third Figure—the Major Premiss must be affirmative, and the Conclusion particular.

Applying these results to the Second Figure, our Major Premiss must be either A or E, since it is universal. Let us inquire which of the four possible Minor Premisses which we might supply to each of these propositions could be contradicted by the Conclusion of the Reduct Syllogism. Let the Major Premiss be A. If the Reduct is to be a legitimate Syllogism in the First Figure, it must be either Barbara or Darii—AAA or

AII.* If its Conclusion be A, this conflicts with the truth of two possible Minor Premisses (of the Reducend), E and O, being the Contrary of the former and the Contradictory of the latter; but if the Conclusion of the Reduct be I, the only Minor Premiss which conflicts with it is E. The Minor Premiss of the Reducend is, therefore, either E or O. We can likewise determine the Conclusion of the Reducend, for the Minor Premiss of the Reduct is its Contradictory. This Minor Premiss is either A or I; consequently the Conclusion of the Reducend must have been either O or E. We thus obtain the Modes AEE, AEO, and AOO.† But AEO is evidently useless, since the Mode AEE is equally capable of verification. Taking now the Major Premiss E, the Reduct, in order to be a legitimate Syllogism in the First Figure, must be either EAE or EIO, Celarent or Ferio. The Conclusion of the former conflicts with two possible Minor Premisses, A and I; that of the latter with A only. Again, to afford us the Reduct Modes

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* The Mode AAI of the First Figure, though useless, is legitimate, and hence ought, perhaps, to be included here, and so of the Mode EAO below. It will be found on examination, however, that their admission would not enable us to verify any additional modes of the other three Figures by *Reductio ad Impossibile.*

† Why not include the mode AOE? It will be seen hereafter that no mode which violates any of our General Rules of the Syllogism can be verified by *Reductio ad Impossibile.** In the present instance, it is plain that AOE would reduce (*per impossibile*) to AII, the Conclusion of which would not conflict with the Suppressed Minor Premiss O. A similar explanation may be given with regard to some other apparent omissions in our list.

Celarent and Ferio, the Conclusion of the Reducend must have been either O or E, and we thus obtain the following verifiable Modes of the Second Figure, viz., EAE, EAO, EIO, of which again we may reject EAO as useless. We thus arrive at the same list that has been already reached by a different method, viz., Cesare, Camestres, Festino, and Baroko.

Passing now to the Third Figure, the Minor Premiss of the Reducend must be either A or I, and its Conclusion must be either I or O; and we have to inquire what Major Premisses are admissible in each case, in order that the Reduct Conclusion should conflict with the Major Premiss of the Reducend. Let the Minor Premiss be A, then the Reduct Syllogism must be either AAA or EAE—Barbara or Celarent. The Conclusion in the former instance would conflict with either of the Major Premisses (of the Reducend) E or O; in the latter, with either of the Major Premisses A or I. We thus obtain four possible pairs of Premisses for the Reducend, viz., AA, EA, IA, OA; and placing as a Conclusion to each the Contradictory of the introduced Major Premiss of the Reduct, we obtain the four Modes AAI, EAO, IAI, OAO. Next, taking I for the Minor Premiss of the Reduct, the mode must be either Darii or Ferio—AII or EIO. In order that the former of these Conclusions should confiict with the Suppressed Major Premiss of the Reducend, that Premiss must have been E; in order that the latter should do so, it must have been A. We thus get for the Reducend Syllogism the pairs of Premisses EI and AI; and supplying, as Conclusion to

each, the Contradictory of the introduced Major Premiss of the Reduct, we find that the Reducend mode was EIO in the one case, and AII in the other. The modes of the Third Figure which can be verified by *Reductio ad Impossibile* are, therefore, AAI, EAO, IAI, OAO, EIO, and AII, or those named Darapti, Felapton, Disamis, Bokardo, Ferison, and Datisi, whose validity has been already established otherwise.*

Thus the validity of all the modes of the Second and Third Figures, which have been recognised as legitimate, can be proved by *Reductio ad Impossibile;* but Aristotle and his followers preferred to employ Ostensive Reduc-

* Aristotle's Dictum does not prove the applicability of the General Rules laid down in the 5th Chapter to any Syllogism which is not in the First Figure, and therefore I could not take these rules for granted, as applying to Syllogisms in the Second or Third Figures in the above discussion. It will be shown hereafter, however, that no mode which violates any of them can be verified by *Reductio ad Impossibile.* Another remark may, perhaps, occur to the reader. It is admitted that, in forming the Reduct Syllogism, I might have started from the Sub-contrary, instead of the Contradictory, of the Conclusion of the Reducend; yet I have only examined the cases in which the Contradictory has been started from. But the fact is, that if the Sub-contrary be started from, a result can only be obtained when the Reducend mode is useless. For the Sub-contraries of either I or O (the only propositions which have Sub-contraries) are the Contradictories of A and E respectively; and if any Syllogism with I or O for its Conclusion could be verified by *Reductio ad Impossibile*, commencing with the substitution of the Sub-contrary of the Conclusion for the Suppressed Premiss, a corresponding Reducend mode, in which the Reducend Conclusion A or E is drawn from the same Premisses, could be equally verified by substituting the Contradictory of *this* Conclusion—the Reduct Syllogism being the same in both cases.

tion wherever it was possible to do so. The only two instances in which it failed were Baroko of the Second Figure and Bokardo of the Third. For it has been remarked that O cannot be a Premiss of a legitimate Syllogism in the First Figure, and that the difficulty cannot be removed by conversion, since O is inconvertible. Hence, too, in applying *Reductio ad Impossibile* to a mode in which O occurs, we cannot retain the Premiss O, since it could not be employed as a Premiss in the Reduct which is to be in the First Figure. It may here be added that, assuming the General Rules already laid down, it can be proved that if O be a Premiss in a legitimate Syllogism, the Syllogism must be either AOO of the Second Figure or OAO of the Third. For this O must be either the Major or the Minor Premiss. Let it be the Major. Then the Minor Premiss must be A, or else we should have either two negative or two particular Premisses. But the mode being negative, the Major Term must be universal in the Major Premiss, in order to avoid an illicit process of that term. Hence the Major Term must be the predicate (not the subject) of the Major Premiss O, which consequently has the Middle Term for its subject. But this subject being particular, the Middle Term must be universal in the Minor Premiss, to avoid an Undistributed Middle; and, in order to be universal, it must be the subject (not the predicate) of the Minor Premiss A. Hence the Middle Term is the subject of both Premisses and the Figure is the Third. Again, let O be the Minor Premiss: then the Major Premiss must be A, or else we should have either two

negative or two particular Premisses. The Conclusion again must be negative, and therefore the Major Term is universal in it, and consequently in the Major Premiss A, where it must be the subject. The Middle Term is, therefore, the predicate of A, where it is particular. Hence, to avoid an Undistributed Middle, it must be universal in the Minor Premiss O, where, consequently, it is the predicate. The figure is thus the Second.

As *Reductio ad Impossibile* was introduced for the sake of proving the validity of Baroko and Bokardo, with respect to which modes Ostensive Reduction failed, it may here be desirable to give these Reductions in the Symbolical Form.

I.—*Baroko.*

Reducend.	Reduct.
Every P is M.	Every P is M.
Some S is not M.	Every S is P.
Some S is not P.	Every S is M.

II.—*Bokardo.*

Reducend.	Reduct.
Some M is not P.	Every S is P.
Every M is S.	Every M is S.
Some S is not P.	Every M is P.

It will be seen that the Conclusion of the Reduct is in each case the Contradictory of the Suppressed Premiss.

It only remains to apply *Reductio ad Impossibile* to the modes of the Fourth Figure. Here we are met by

one difficulty:—Whichever Premiss we suppress, the order of terms in it will be wrong, whereas the order will be right in the Contradictory of the Conclusion which supplies its place. It will, therefore, be right in the Conclusion of the Reduct, since the Reduct is in the First Figure.* Hence the terms of the Conclusion of the Reduct are in a different order from those of the Suppressed Premiss. But this does not prevent the method from applying, since if we can *convert* the Conclusion of the Reduct into the Contradictory (or Contrary) of the Suppressed Premiss, or *vice versâ*, both cannot be simultaneously true, which is all that we require to establish. The Reduct Syllogism will here be in the First Figure whichever Premiss we suppress, but some cautions must be observed in order to make it a *legitimate* Syllogism in the First Figure. Thus, if the Conclusion of the Reducend is universal, we must suppress the Minor Premiss, for the Contradictory of that Conclusion, being particular, could not be the Major Premiss of a legitimate Syllogism in the First Figure. And in that case the Conclusion of the Reducend must have been negative, since its Contradictory must be

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* The *Extreme* of the Suppressed Premiss is the *same* extreme in both Syllogisms. In the Fourth Figure this extreme is in the *wrong* place in the Suppressed Premiss, but is in the *right* place in the Substituted Premiss (the Contradictory of the Conclusion of the Reducend). But it is in the *same* place in the Conclusion of the Reduct as in this Substituted Premiss, since the Reduct is in the First Figure. This is what is meant by saying that in the Conclusion of the Reduct this Term is in its right place.

affirmative in order to become a Minor Premiss in the First Figure. Again, if the Minor Premiss of the Reducend be negative, it must be suppressed; in which case, since the Conclusion must be negative also, its Contradictory will supply an affirmative Minor Premiss for the Reduct. Further, if the Major Premiss be particular, it must be suppressed for the like reason; and as the Conclusion of the Reducend must in that case be particular also, its Contradictory will furnish an universal Major Premiss for the Reduct. We cannot indeed arrive at the Special Rules of the Fourth Figure in the same simple manner with those of the Second and Third Figures, but we can see already that to apply *Reductio ad Impossibile* to a mode of the Fourth Figure, it is necessary either that we should have an universal Major Premiss *and* a negative Conclusion, or an affirmative Minor Premiss *and* a particular Conclusion. Now, the modes of the Fourth Figure, which have an universal Major Premiss and a negative Conclusion, are AEE, EAE, EAO, EIO, and AOO, and those which have an affirmative Minor Premiss and a particular Conclusion are AAI, IAI, EAO, OAO, AII, and EIO.* The former affords a list of the modes of

* It may be objected that in drawing out this list, as well as on some previous occasions, I have assumed some of the General Rules which have not been proved on Aristotle's principles, viz., that from two negative Premisses nothing follows: that from two affirmative Premisses a negative Conclusion cannot follow: that if one Premiss be negative, the Conclusion is negative: that if one Premiss be particular, the Conclusion is particular: and that from two particular Premisses nothing follows. But the Dictum proves immediately that

the Fourth Figure, whose Reduction appears to be feasible by suppressing the Minor Premiss, the latter of those whose Reduction appears to be feasible by suppressing the Major Premiss; and the Modes EAO and EIO, which are comprised in both lists, seem to be reducible in both ways. But further examination reduces the number. Recollecting that the order of terms in the Conclusion of the Reduct is now different from their order in the Suppressed Premiss, if one of these be A and the other O, we have arrived not at Contradictories but at propositions which are *consistent* with each other—propositions both of which may be true at the same time. The proposition All B is C is not only consistent with the proposition Some C is not B, but both will in fact be simultaneously true whenever the for-

all these Rules are true of Syllogisms *in the First Figure,* and this enables us to prove that if they are violated in any other Figure, *Reductio ad Impossibile* becomes inapplicable. For, 1st. Let both Premisses of the Reducend be negative. The Retained Premiss being negative, the Conclusion of the Reduct Syllogism (which is in the First Figure) will be negative. But the Suppressed Premiss is also negative, and, between two negative propositions having the same terms (whether in the same order or not), there is no inconsistency. 2nd. Let one Premiss of the Reducend be negative and its Conclusion be affirmative. Then, if the negative Premiss be retained, the Reduct Syllogism will have two negative Premisses, which is inadmissible in the First Figure; if it be suppressed, the Reduct Syllogism will have one negative Premiss, and, therefore (being in the First Figure), a negative Conclusion, which cannot be inconsistent with the Suppressed Premiss, the latter being also negative. 3rd. Let the Conclusion of the Reducend be negative, while both its Premisses are affirmative. Then the Reduct will have two affirmative Premisses and (being in the First Figure) an affirmative Conclusion, which cannot be inconsistent with the Suppressed Premiss, which is also affirmative.

mer is so unless the classes B and C are exactly coextensive. And if the Suppressed Premiss be O, it is impossible that the Conclusion of the Reduct should be inconsistent with it, for every one of the four propositions A E I and O is consistent with the truth of the proposition O, with the order of the terms reversed. In like manner, if the Conclusion of the Reduct Syllogism be O, it cannot conflict with the truth of the Suppressed Premiss, whatever that Premiss may have been; and hence, if the Reduct Syllogism be in the mode *Ferio*, our Reduction has failed to accomplish anything. Turning now to the list of modes of the Fourth Figure, to which *Reductio ad Impossibile* seems applicable, by suppressing the Minor Premiss, viz., AEE, AEO, EAE, EAO, EIO, and AOO, we

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4th. Let one Premiss of the Reducend be particular and its Conclusion be universal. Then, if the particular Premiss be retained, the Reduct will have two particular Premisses, which is inadmissible in the First Figure; but if it be suppressed, the Reduct will have one particular Premiss, and therefore (being in the First Figure) a particular Conclusion, which cannot be inconsistent with the Suppressed Premiss, since the latter is also particular. (Two Sub-contraries may both be simultaneously true, and the same thing is evident of the propositions I and O with an opposite order of terms, which, indeed, become Sub-contraries by simply converting I.) 5th. Let the Reducend have two particular Premisses. The Retained Premiss being particular, and the Reduct being in the First Figure, its Conclusion must be particular; but the Suppressed Premiss is also particular, and hence there is no inconsistency. I have assumed in each case that the Reduct Syllogism is a valid one, leading to *some* Conclusion. Where it is an invalid Syllogism, the Reduction still more obviously fails. It is only necessary to bear in mind that two Contradictory Propositions differ both in quantity and quality, to see the force of these proofs at a glance.

must reject the last because the Suppressed Premiss will be O, and the third because the Reduct Syllogism will be in Ferio. There remain, therefore, AEE, EAO, and EIO. Similarly, examining the list of those whose reduction appears possible by suppressing the Major Premiss, viz., AAI, IAI, EAO, OAO, AII, and EIO, we reject the fourth because the Suppressed Premiss will be O, and the fifth because the Reduct Syllogism will be in Ferio: and as EAO and EIO are included in both lists, we have reduced the number to the five already recognised as valid, with the addition of the useless mode AEO. A short examination will satisfy the reader that in each of these five, or rather six, modes, the Conclusion of the Reduct Syllogism is inconsistent with the Suppressed Premiss.

But it may be asked, was this lengthy process necessary even on Aristotle's principles? And if we admit Obversion among the processes which may be employed in Ostensive Reduction, the reply must be in the negative. *Reductio ad Impossibile* was only resorted to in order to establish the validity of the modes Baroko and Bokardo, both of which can be reduced ostensively to the First Figure by admitting Obversion among the processes employed in Ostensive Reduction. These Modes, expressed symbolically, are as follows:—

I.—*Baroko.*

Every P is M.
Some S is not M.
Some S is not P.

II.—*Bokardo.*

Some M is not P.
Every M is S.
Some S is not P.

In the first of these cases we substitute for both Premisses their Obverses, and thus obtain:—

No P is not-M.
Some S is not-M.

These Premisses are evidently in the mode Festino of the Second Figure, and can be reduced to Ferio of the First Figure (giving us the old Conclusion), by simply converting the Major Premiss No P is not-M. If we use the letter K as the sign of Obversion,* this mode might be called *Faksoko.* In the second case we need only substitute for the Major Premiss its Obverse in order to obtain

Some M is not-P.
Every M is S,

which Premisses are evidently in the mode Disamis of the Third Figure, and can be reduced to Darii of the First Figure by transposing the Premisses, and simply converting the old Major Premiss, which will then

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* The letter *K* in the names Baroko and Bokardo denoted that the Premiss indicated by the preceding vowel was to be suppressed, and the Contradictory of the Conclusion substituted for it. The mode of the resulting Syllogism (in the First Figure) was indicated by the initial letter, being in both instances Barbara.

have become the Minor Premiss. Performing this operation:

Every M is S.
Some not-P is M.
Some not-P is S.

In order to derive our original Conclusion from this Conclusion, we must simply convert it, and then substitute for the simple Converse its Obverse. The whole process would, therefore, be described by the name *Dokamosk*, the last two letters implying that the new Conclusion is to be simply converted, and then the Obverse of this simple Converse substituted for it, in order to arrive at the original Conclusion O.*

As Aristotle did not recognise the Fourth Figure, it may be desirable to state here how its legitimate modes were dealt with in the Aristotelian System. They were treated as Indirect Modes of the First Figure. We have seen that though, for the sake of uniformity, we write the Major Premiss first in Logic, there is no necessity for so doing. Now, the Logicians who did not recognise the Fourth Figure treated Bramantip as Barbara, with its Minor Premiss written first *and its Conclusion converted*, Camenes as Celarent, subjected to the

* The initial letter of these names sufficiently indicates the change which takes place in the vowels on performing the processes; but if it is desired to express this more fully, the vowel representing the changed propositional Form may be written after that representing the original one, and the two modes in question may thus be called *Faeksoiko* and *Doikamoisk*. In these names the letter K follows the Obverse proposition, not the Obvertend.

same process, and Dimaris as Darii similarly treated. When, however, they came to deal with Fesapo and Fresison, this explanation failed, since the pairs of Premisses AE and IE are not valid in the First Figure. But the Logicians in question said that, though these pairs of Premisses led to no *direct* Conclusion in the First Figure, they led to an *indirect* Conclusion—a Conclusion of which the Major Term was the subject, and the Minor Term the predicate. This is rather a remarkable instance of denying in words what is admitted in fact. The five Indirect Modes of the First Figure, according to these Logicians, were Baralip, Celantes, Dabitis, Fapesmo, and Frisesmo,* the identity of which list with the recognised Modes of the Fourth Figure (save that the Minor Premiss of the Fourth Figure is invariably treated as the Major Premiss and *vice versâ*, which renders it necessary for the letter *m* to appear in one list wherever it does *not* appear in the other) is immediately evident.† Sir William Hamilton, however, proposes to abolish the Fourth Figure more effectually by supplying *direct* Conclusions to those modes of the First Figure which, according to previous rejectors of the Fourth Figure, led only to indirect ones. AE and IE, according to him, are legitimate pairs of Premisses

* These Logicians admitted IEO into their list of legitimate Syllogisms because in the First Figure it afforded an *indirect* Conclusion.

† Of course these Logicians only recognised the Special Rules of the First Figure as applying to Modes in which *direct* Conclusions were to be attained. *Indirect* Conclusions might be arrived at, though the Minor Premiss was negative and the Major Premiss particular.

in the First Figure, each of them leading to the direct Conclusion η, the simple Conversion of which Conclusion affords the so-called mode of the Fourth Figure. And, if his System be adopted, every mode of the Fourth Figure can undoubtedly be represented as an indirect mode of the First, while every mode of the First Figure might be equally described as an indirect mode of the Fourth. But I have already stated my reasons for not accepting Sir William Hamilton's Theory.*

It has been already mentioned that some Logicians have endeavoured to establish directly the validity of

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* Before dismissing the subject of *Reductio ad Impossibile*, it may be remarked that, assuming that an Undistributed Middle or an Illicit Process of either Term is inadmissible in the First Figure (which sufficiently appears if we admit Aristotle's Dictum as the sole test of the validity of a Syllogism in that Figure), it may be easily proved that a Syllogism in any other Figure which contains either of these defects cannot be verified by *Reductio ad Impossibile*. In establishing this, it is only necessary to bear in mind that two propositions are always consistent with each other if the same term is particular in both of them, and that in two Contradictory propositions both terms differ in quantity. Let us suppose, for instance, that the Reducend contains an Illicit Process of the Major Term. That term, being universal in the Conclusion of the Reducend, is particular in its Contradictory. Hence, if we retain the Major Premiss, the new Middle Term will be undistributed (for it must be particular in the Retained Major Premiss, if the Reducend contains an Illicit Process of the Major Term). Therefore we must suppress the Major Premiss. The old Major Term is particular in it. But it is also particular in the Contradictory of the Conclusion of the Reducend, and hence it will be particular in the Conclusion of the Reduct, since the Reduct is in the First Figure. Being therefore particular both in the Suppressed Premiss and in the Conclusion of the Reduct, there is no inconsistency between them.

Syllogisms in the Second, Third, and Fourth Figures, by assuming Dicta for each of these Figures which are supposed to be equally self-evident with that of Aristotle, which latter directly establishes the validity of a Syllogism in the First Figure only. Thus the validity of a Syllogism in the Second Figure can be deduced from the Dictum, *If one term is contained in, and another excluded from, the same third term, they are mutually excluded.* For the Third Figure the following Dictum has been proposed: *Two terms which contain a common part partly agree, and if one contains a part which the other does not, they partly differ;* while the Fourth Figure has got a more cumbrous Dictum, with two branches: 1st. *If no C is B, no B is this or that C;* 2nd. *If C is or is not this or that B, there are Bs which are or are not this or that C.* The object of these Dicta is, of course, to supersede the necessity of the Reductions which we have been considering.

Aristotle was satisfied with reducing the modes of other Figures to any mode of the First, since all the modes of the First Figure fall equally within the terms of the Dictum. Had it been necessary to do so, however, the entire number might have been reduced to Barbara (or Celarent) alone. Celarent reduces to Barbara by substituting for the Major Premiss its Obverse, and the original Conclusion is obtained by Obverting the Conclusion of the new Syllogism in Barbara. Darii reduces to Ferio by a similar process: Ferio becomes Festino by simply Converting its Major Premiss, and Festino can be verified by *Reductio ad Impossibile*

when the Reduct Syllogism will be Celarent, which is, as we have seen, reducible to Barbara. Barbara could also be reduced to Celarent, and all valid Syllogistic modes could thus be verified, if the validity of the mode Celarent were conceded. Reductions of this kind, however, are of no value, except perhaps as exhibiting the coherence of the various parts of the Logical Theory of the Syllogism: and the student will have already observed how frequently the same results may be arrived at by the employment of apparently different means.

CHAPTER XVII.

SOME FURTHER REMARKS ON THE SYLLOGISM.

AMONG the rules sometimes laid down by Logicians are, 1st, *That from truth nothing but truth can follow*, and 2nd, *That from falsehood truth may follow*. The first of these rules means that from one or more true propositions (treated as true), no false proposition can be deduced. This again amounts to stating that Logic is concerned only with *conclusive* inferences, since if any kind of inference which is not absolutely conclusive be admitted, it must sometimes be *possible* to deduce a false Conclusion from true Premisses. The second rule means that from one or more false propositions treated as true (if we *treat* them as false, we substitute their Contradictories

for them, and then reason from true Premisses as in the case already considered) we may deduce a true Conclusion. This is always possible except in the case of those immediate inferences which preserve the *whole* of the assertion comprised in the false proposition with which we set out; as is done, for example, by the simple Conversion of E or I. In these cases a second simple Conversion restores the original proposition, and, as it is false, the proposition *which leads to it by Conversion* must be false likewise. But if any part of a false assertion is dropped, the remaining part may be true. *The falsity of one or both of the Premisses of a legitimate Syllogism never implies the falsity of the Conclusion:* though some persons are very apt to conclude that a proposition is false as soon as they find it defended on untenable grounds. That the falsity of *both* Premisses will not enable us to infer the falsity of the Conclusion is pretty obvious. To assume the falsity of any proposition is exactly the same thing as to assume the truth of its Contradictory. Now, if in any affirmative mode we substitute for both Premisses their Contradictories, we obtain two negative Premisses, from which no Conclusion follows; while, if we perform the same operation on a negative mode, the new Syllogism will also have a negative Premiss (the Contradictory of the affirmative Premiss of the former Syllogism), and the Conclusion (if any) will be negative, in which case it cannot be inconsistent with the negative Conclusion of the original Syllogism. It is somewhat more difficult to prove the same thing where one Premiss only is false;

and perhaps the best way of doing so is to show that from one Premiss and the Conclusion of a legitimate Syllogism, taken as Premisses, we can never legitimately deduce the *other* Premiss as a Conclusion; or, in other words, *the mutual substitution of a Conclusion and Premiss is never legitimate.* For, if the truth of the Conclusion and one Premiss does not imply the truth of the other Premiss, it is evident that this other Premiss may be false when the Conclusion is true.

We have seen that the effect of substituting a Conclusion for a Premiss on the position of the terms is the same as that of substituting its Contradictory in *Reductio ad Impossibile.* The extreme of the Retained Premiss becomes the new Middle Term. If then we substitute the Conclusion for either Premiss, and the new Syllogism is to be valid, this Term must be universal in the Retained Premiss: for it must be *once* at least universal in the new Syllogism, and it could not have been universal in the Conclusion of the old Syllogism unless it was so in the Retained Premiss. Further, the Retained Premiss cannot be either negative or particular: for in that case the Conclusion of the old Syllogism must have been negative or particular also, and the new Syllogism would thus have either two negative Premisses or two particular Premisses. The Retained Premiss must, therefore, be A, and the old Extreme being universal in it must be its subject. The old Middle is, therefore, the predicate of the Retained Premiss A, where it is particular: it must, consequently, have been universal in the Suppressed Premiss, or else the old Syllogism

would have contained an Undistributed Middle. But the predicate of the Retained Premiss A is an Extreme in the new Syllogism, and must be particular in the Conclusion of the new Syllogism, otherwise the new Syllogism will contain an Illicit Process of that Extreme. Since, therefore, this term is particular in the new Conclusion, and universal in the Suppressed Premiss, the new Conclusion cannot be the Suppressed Premiss; nor can it be converted into the Suppressed Premiss, since Conversion never increases the quantity of a term. It is equally clear that this defect cannot be removed by transposing the Premisses of the new Syllogism. Hence one Premiss may always be false when both the Conclusion and the other Premiss are true, and therefore the falsity of one Premiss never implies the falsity of the Conclusion.* The rule, that from falsehood truth may

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* The same thing otherwise appears thus:—If the falsity of one Premiss (the other being supposed true) implies that of the Conclusion, it must be possible from the Contradictory of this Premiss and the other Premiss to deduce a Conclusion inconsistent with our former one. For this purpose the Retained Premiss must be A; for, as two Contradictories differ both in quantity and quality, if the Retained Premiss were particular or negative, *one* of the two Syllogisms would have two particular or two negative Premisses. And the subject of this Retained Premiss A must be the Middle Term (of both Syllogisms): for the quantities of *both* terms are different in any proposition and its Contradictory; and if the Middle Term were particular in the Retained Premiss, it would therefore be Undistributed in one of the two Syllogisms. The predicate of A is, consequently, an Extreme in both Syllogisms; and being particular in the Retained Premiss A, it must be particular in the Conclusions of *both* Syllogisms; and the same term being particular in both Conclusions, they are not inconsistent with each other.

follow, thus holds good of all Syllogisms. It likewise appears that since from one Premiss of a Syllogism and its Conclusion (taken as Premisses) we can never arrive at the other Premiss as a Conclusion, there cannot be more than one way of putting together three given propositions so as to form a valid Syllogism. It would be easy to give rules for constructing a legitimate Syllogism from three propositions capable of forming one, but in practice it is never necessary to do so.* If a reasoner expresses himself so carelessly that it is impossible to tell what are his Premisses and what is his Conclusion, it is not likely that the three propositions which he enounces will prove capable of forming a legitimate Syllogism while, if they should happen to do so, it is as probable that some other order was intended as the correct one. The context will throw more light on what the reasoning was intended to be than any rules of Logic.

The proof in the text, however, has the advantage of showing that from the same three propositions it is impossible to construct more than one legitimate Syllogism.

* The simplest rule is this:—If two of the propositions are universal and the third particular, the former are the Premisses and the latter is the Conclusion. If not, it will be found that two of the terms occur twice with the same quantity, while the third term is once universal and once particular. In this case the former are the Extremes and the latter is the Middle Term. For if the Middle Term occurs twice with the same quantity (*i. e.* is twice universal), or if either of the Extremes changes its quantity (*i. e.* is universal in its Premiss and particular in the Conclusion), the Premisses contain two more Universal Terms than the Conclusion, and consequently the Syllogism has two universal Premisses and a particular Conclusion. This has been already proved.

Sir William Hamilton's system has met with so much acceptance, that though I have not adopted it, its effects on the Theory of the Syllogism may be briefly pointed out. The Axioms laid down in Chapter V. are as applicable to propositions in Sir William Hamilton's Forms as in the ordinary ones, and therefore in his system, as well as in that of Aristotle, it will be true that from two negative Premisses nothing follows, that if one Premiss be negative, the Conclusion (if any) will be negative, and that from two affirmative Premisses there cannot be a negative Conclusion. It is likewise equally true on his system that a Syllogism is invalid, if it contains an Undistributed Middle, or an Illicit Process of either Extreme. But in applying these latter rules, we must now proceed very differently from our former course; for we formerly assumed that the predicate of an affirmative proposition is particular, and that of a negative proposition universal, neither of which assumptions holds good in the system which we are now considering. Confining our attention, for the present, to Pairs of Premisses, we may throw the possibility of an Illicit Process out of account altogether; for if either term be particular in its Premiss, we can always draw a Conclusion in which it is likewise particular. So, too, of course we can always draw a Conclusion which will conform to the rules against an affirmative Conclusion from a negative Premiss, or a negative Conclusion from two affirmative Premisses. Hence the only faults which require to be guarded against *in Pairs of Premisses* on this sys-

tem are—1, Negative Premisses; and 2, Undistributed Middle. Taking each of Hamilton's 8 Forms successively as Major Premiss, and supplying to it each of the 8 possible Minor Premisses, the total number of possible Pairs of Premisses will be 64. But we must first strike out the Pairs in which any one of the four negatives E, O, η, ω is followed by a second negative, of which there will be 16. Again, whether the Middle Term is the subject or predicate of the Major Premiss, it will be particular in four out of the eight possible Major Premisses U, A, Y, I, E, O, η, ω; while it will be likewise particular in four out of the eight possible Minors. Hence if any of the four former is followed by any of the four latter, there will be an Undistributed Middle. This fault, therefore, will also occur in 16 Pairs of Premisses; but referring back to the Pairs of negative Premisses it will be found, that whether the Middle Term is the subject or predicate of the Major Premiss, two of these negative Majors will render it particular in that Premiss, and two of the negative Minors will render it particular in the Minor Premiss also. There will thus be four Pairs of negative Premisses containing an Undistributed Middle, and, consequently, after striking out of our 64 Pairs of Premisses the 16 negative Pairs, we have only 12 *additional* Pairs of Premisses to strike out for Undistributed Middle. Deducting these 28 from the 64 we obtain 36, which is the number of Legitimate Modes, according to Sir W. Hamilton, *in each figure.* In fact, since, with Hamilton, all Conversion is reducible to Simple

Conversion (or writing the same equation the opposite way), and since a Pair of Premisses in any Figure can be turned into an exactly equivalent Pair in any other Figure, by simply converting one or both of the Premisses, it follows that to every legitimate Pair in any Figure will correspond a precisely equivalent Pair in any other. The four Syllogisms thus formed are really identical. They are in fact the same Syllogism written in four different ways. Each Pair of Premisses has its own Conclusion, which may of course be varied by Conversion, and in most cases (unless we take *Some* to mean Some only) by Subalternation* also. But all such Conclusions depend on one ultimate one. From two simple equations, each expressing one of the Extremes in terms of the Middle, we cannot by any difference in the mode of writing these equations deduce two really distinct relations between the Extremes themselves; and no Categorical proposition in this system represents an equation of a higher degree than the first.

The consideration of these mutually dependent Conclusions, however, suggests a possible reduction of our list. The modes of the Fourth Figure in this system, as already stated, may be all explained as modes of the First Figure, with the Minor Premiss written first, and

* Subalternation can be practised more extensively than in the ordinary system, since we can reduce the quantity of the predicate as well as that of the subject; or if both terms are universal, we can reduce the quantity of both. In this, however, as in the ordinary system, we can never increase the quantity of a term.

the Conclusion converted by writing the equation with its terms inverted: while the modes of the First Figure may be derived from those of the Fourth in the same way. Sir William Hamilton accordingly abolishes the Fourth Figure, and treats its modes as indirect modes of the First. But again in this system, as the Conclusion always admits of being simply converted, so the Simple Converse of the Conclusion can always be treated as the Conclusion of the new Syllogism formed by transposing the Premisses of the former one. This transposition of Premisses in the First Figure would indeed lead to the formation of a Syllogism in the Fourth Figure, which Hamilton does not recognise; but in the Second and Third Figures, the new Syllogism is in the same Figure with the old, and the number of really distinct modes in each of these Figures is thus reduced from 36 to 18. Indeed Hamilton states that in the Second and Third Figures, since both Extremes bear the same relation (as respects subject and predicate) to the Middle, there is no determinate Major or Minor Premiss, and there are always two Indifferent Conclusions. This being so, it seems erroneous to treat as distinct two modes in each of which the same two Premisses lead to the same two Indifferent Conclusions; and we must therefore, I apprehend, fix the number of legitimate modes in each of these Figures at 18. Something like this reduction, though on a more limited scale, is possible under the ordinary system. From two Premisses of the forms A and E, with the Middle Term as predicate of both, we can draw

two Conclusions both of the form E, each of them deducible from the other by Simple Conversion, one of which is supposed to be deduced *directly* from the Premisses when we call A the Major Premiss, while when the other is supposed to be the direct Conclusion, we confer that title on E. The mode is called *Cesare* in the one case, and *Camestres* in the other, but the two Syllogisms are really identical. The same observation is true of the Pair of Premisses A and I, when the Middle Term is the subject of both, so that the modes designated *Disamis* and *Datisi* are not really distinct. If we reduce them ostensively to the First Figure (and the same remark applies to Cesare and Camestres), we obtain the same Syllogism in both cases, the only difference being that in one case the Conclusion of the Reduct is the Conclusion of the Reducend, while in the other it leads to that Conclusion by Simple Conversion. One more remark, which has been partly anticipated, may be made on the Hamiltonian list. Including the Fourth Figure among those which may be legitimately employed, we have seen that the effect of simply converting (which in this system is always possible) one or both of the Premisses of any Syllogism is to give us a precisely equivalent Syllogism in a different Figure. This proves that, if we choose to do so, we can get rid of all Syllogisms in which either η or Y occurs as a Premiss, and yet succeed in drawing the same Conclusion from substantially the same Premisses. If we convert Y, whenever it occurs, into A, and η, whenever it

occurs, into O, it is clear that any Conclusion that could be validly drawn from the old Premisses will follow with equal validity from the new.* The introduction of the forms Y and η, therefore, as already noticed, never enables us to arrive at a Conclusion which could not have equally been inferred from substantially the same assertions expressed in the forms recognised by Logicians generally; and as Sir William Hamilton fully recognises the identity of the Syllogism under the four possible variations of form (the four figures), he must admit that his Forms Y and η do not lead us to any modes of reasoning that are new in substance. The real addition which Hamilton has made to the ordinary table of Syllogistic Modes consists of those into which U or ω enter, whether as Premisses or Conclusions, and there is no doubt that the assertions conveyed by either of these propositions cannot be expressed by any *single* proposition of the ordinary form; but I have endeavoured to prove that this arises merely from the fact that these propositions are complex or compound.†

Some Logical Treatises deal at considerable length with the Reduction of a Sorites into Syllogisms, but it has been remarked that such reduction is not requisite,

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* And if Y or η occurs as a Conclusion we can replace it by A or O, as the case may be, altering the order of the terms, and in this way deduce substantially the same Conclusion from the same Premisses.

† Of Hamilton's 36 Modes, 24 are Negative, and 12 Affirmative. In fact it will be found, that of the 64 possible Pairs of Premisses there are 16 pairs of Affirmatives, 16 pairs of Negatives, 16 pairs with an Affirmative Major and a Negative Minor, and 16 pairs with a Negative

in order to see the validity of the Sorites. In general, if the Reduction is attempted, we must reduce the Sorites to a series of Syllogisms in the First Figure; for if any other Figure be admitted, we shall usually either reach no final Conclusion, or else one different from that which we seek to prove, and not leading to it by Subalternation, Conversion, or any other Logical process. This is evident, if the Conclusion of the Sorites is A; for it is only in the First Figure that we can arrive at A as the Conclusion of a Syllogism, and it cannot be inferred from any other Proposition (in one of the four recognised forms) by any legitimate means (except Obversion). If the Sorites is of the ordinary form, its second Premiss must be made the Major Premiss of the first Syllogism, and its first Premiss the Minor, in order to obtain the First Figure. A second Syllogism will next be formed, composed of the Conclusion of *this* Syllogism and the third Premiss of the Sorites; then a third Syllogism, composed of the Conclusion of the second Syllogism and the fourth Premiss of the Sorites, and so on till we reach the end. In order to keep these successive Syllogisms in the First Figure, we must take care to make the Conclusion of each intermedi-

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Major and an Affirmative Minor. Again, of the 16 pairs which are bad for Undistributed Middle, 4 are pairs of Affirmatives, 8 are pairs consisting of one Affirmative and one Negative, and 4 are pairs of Negatives. The legitimate Affirmative Pairs are thus reduced from 16 to 12, and the pairs consisting of one Affirmative and one Negative from 32 to 24.

ate Syllogism the Minor Premiss of the following Syllogism. For the predicate of each Premiss of the Sorites is the subject of the following one. When, therefore, in our series of Syllogisms we reach any given Premiss, the subject of that Premiss has formed an extreme in the preceding Syllogism (for the preceding Premiss of the Sorites was one of the Premisses of the preceding Syllogism; and this term having for the first time appeared as the predicate of that Premiss, could not have occurred more than once in the preceding Syllogism, and must, therefore, have been an extreme in it), and therefore appeared in its Conclusion. Hence, in the Syllogism which we are now forming (consisting of the Conclusion of the preceding Syllogism and the Premiss of the Sorites that we have now reached) this term will be the Middle Term; and since it is the subject of the Sorites-Premiss, that Premiss must be made the Major Premiss of the Syllogism which we are now forming, and of course the Conclusion of the preceding Syllogism will be the Minor Premiss of it. It has not indeed been proved that the term in question is the predicate of the Conclusion of the preceding Syllogism; but it will be so if the preceding Syllogism was in the First Figure, since in *that* Figure the terms of the Conclusion occupy the same position in the Conclusion that they occupied when they previously occurred in the Premisses. Now, the term in question was the predicate of the preceding Premiss of the Sorites; therefore it was also the predicate of the Conclusion of the preceding Syllogism, if that Syllo-

gism was in the First Figure. We can now prove the Rules of the Sorites, if we assume that a Sorites is always reducible to a series of Syllogisms in the First Figure. For the Conclusions of all the Syllogisms into which it is resolved will be Minor Premisses of subsequent Syllogisms in the First Figure, and therefore affirmative. But if a negative Premiss entered into the Sorites at any stage earlier than the last, the Conclusion of the Syllogism into which this Premiss entered would be negative. Hence no negative Premiss is admissible until the final Syllogism, and therefore the negative Premiss, if any, must be the last Premiss of the Sorites. Again, all the Premisses of the Sorites, except the first, become the Major Premisses of successive Syllogisms in the First Figure. Hence they must all be universal. Analogous rules may be easily framed for the Goclenian Sorites.*

The view which has been taken of a Dilemma is not universally adopted either in popular language or by Logicians. We sometimes, in popular works, hear of a person being on the Horns of a Dilemma when he has two disagreeable alternatives to select from. Sup-

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* The rules in question are not absolutely proved, since it has not been shown that no Sorites can be legitimate, unless it is resolvable (without converting any of its Premisses, or any of the Intermediate Conclusions arrived at) into a series of Syllogisms in the First Figure. It seems possible that it might, notwithstanding, be resolved into a series of Syllogisms in some of the other Figures. And, in fact, no such hard and fast rules seem capable of being laid down. A Syllogism in the Fourth Figure may be regarded as a Sorites of two Premisses, and it is

posing, however, that he has been placed in this difficulty by the result of a train of reasoning, it is clear that this train of reasoning must have led to a Conclusion of the form Either B is C or D is F, and the person in question is placed in the difficulty, because he must accept either member of this Disjunctive Proposition. The train of reasoning would therefore resolve itself into one or more Dilemmas, in the extended sense of that term mentioned at p. 71—such a Dilemma being the only kind of Syllogism in which the Conclusion is Disjunctive. But in popular language and in some Logical works the term Dilemma is only used when both members of this Disjunctive Conclusion are unacceptable to the person or persons to whom it is addressed. In a Treatise on Logic, however, such a distinction as this ought not to be recognised, and the term Dilemma (if used at all) should be employed to designate some particular kind of *argument*, irrespective of the disposition of the persons to whom it is addressed. Archbishop Whately, who defines a Dilemma as *A Syllogism with several antecedents in the Major (Premiss) and a Disjunctive Minor (Premiss)*—which substantially accords with the description I have

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not essential to the validity of such a Syllogism that the first Premiss should be affirmative, or the second universal. The occurrence, however, of a *second* negative or a *second* particular Premiss in a Sorites would be fatal to its validity, since, when we arrived at that stage in any kind of reduction, we should have a Syllogism with two negative or two particular Premisses. This observation applies to the Premisses of the Sorites, not to its Conclusion.

given—seems to me to fail in his reduction of it to the Categorical form. His mode of reduction will be best understood from his example—

If Æschines joined in the public rejoicings, he is inconsistent; if he did not join, he is unpatriotic.
But he either joined or did not join.
Therefore he is either inconsistent or unpatriotic.

Whately (having shown how to reduce an ordinary Hypothetical Syllogism to the Categorical form) proposes to reduce the above to *two* Hypothetical Syllogisms, each with the Major Premiss given above, but one with the Minor Premiss *He did join* and the Conclusion *Therefore he is inconsistent*, and the other with the Minor Premiss *He did not join* and the Conclusion *Therefore he is unpatriotic.* These Syllogisms are so related to each other that though an opponent might deny *either* of the Minor Premisses, he could not deny both, and therefore he must admit *one* of the two Syllogisms in question to be correct, while either of them will answer the reasoner's purpose, namely, to prove that Æschines was in the wrong. Such a reduction seems to me to miss the true character of the reasoning comprised in a Dilemma. It is true that an opponent could not *deny* the Minor Premisses of Whately's two Syllogisms without self-contradiction; but (supposing he was not Æschines himself; and we must take care not to reduce general forms of argumentation to mere arguments *ad hominem*) he might very well reply, that he did not know whether he joined

or not, and, therefore, could not accept either of the Hypothetical Syllogisms as correct. To this Whately could only rejoin, that though his opponent might be uncertain whether Æschines joined or did not join in the public rejoicings, he must admit if the truth were known that he did either the one or the other, and, therefore, he must be *either* inconsistent *or* unpatriotic. But this rejoinder amounts to abandoning the two Hypothetical Syllogisms with Categorical Conclusions, and reverting to the original Dilemma with its Disjunctive Conclusion, which thus remains unreduced. The true mode of reducing it has been indicated already. It is a single argument leading to a Disjunctive Conclusion, not two distinct, though connected, arguments, each leading to a distinct Categorical Conclusion.

Logical reasonings are sometimes called Arguments *ad rem*, as distinguished from other kinds of argument, which are enumerated. Among these the most common is an argument *ad hominem*. Such an argument, however, is easily reduced to the Syllogistic form, as Archbishop Whately has observed. It is always of the following form:—

Mr. So-and-so admits the proposition B is C.

The proposition D is F is a necessary consequence of the proposition B is C.

Therefore Mr. So-and-so is bound to admit that D is F.

This reasoning can be reduced to strict Syllogistic

form with little difficulty; but in *proving* that the proposition D is F is a necessary consequence of the proposition B is C, it may be requisite to introduce a chain of reasoning which will increase the number of Syllogisms to which the entire argument is reducible. And instead of Mr. So-and-so, some political or religious party may be named. Indeed, arguments of the kind are often prefaced by a statement that Mr. So-and-so is the ablest exponent or defender of some particular theory, whence it is inferred that if he can be involved in some inconsistency or absurdity, all other advocates of the theory in question can be involved in the same; whence again it is concluded that these inconsistencies or absurdities are involved in the theory itself. Where this is done there are, in addition to the argument *ad hominem* addressed to Mr. So-and-so, two further arguments—one seeking to extend the inference from Mr. So-and-so to all the advocates of his theory, and the other carrying on the inference to the theory itself: and if either of these further arguments breaks down, the Conclusion which the reasoner really seeks to draw is not established, although the original argument *ad hominem* should prove to be valid.* An argument *ad hominem* admits of various refutations. Mr. So-and-so

* A remarkable instance of this occurs in Mr. Mill's discussion on Mathematical Necessity in the Second Book of his Logic. Few competent judges will now contend that Dr. Whewell's *a priori* theory of Mathematics is as tenable as the prior theory of Kant (which Whewell had evidently failed to master), and the refutation of Whewell leaves Kant almost wholly untouched.

may reply that he does not admit the proposition B is C, or at least that he does not admit it in the sense in which it is used by his opponent. He may deny that the proposition D is F necessarily follows from the proposition B is C, or he may accept the Conclusion D is F and endeavour to show that it involves nothing inconsistent or absurd. Lastly, he has a course open to him which indeed is open in all cases where a Conclusion which seems erroneous is deduced from hitherto admitted Premisses—namely, to say that though he formerly, through haste and insufficient consideration, admitted that B is C, he has now altered his opinion, and maintains the contrary. This course might often be adopted with advantage, but it is not very gratifying to Mr. So-and-so's pride to have recourse to it. Very similar to an argument *ad hominem* is an argument *ad verecundiam*. It is an appeal to some authority which is recognised by the persons to whom the argument is addressed, for example, to the Church, to the founder of a religious sect, to some eminent Philosopher, or to the head of some political party. There are three ways of meeting it. First, it may be denied that the authority appealed to has in fact laid down what is asserted: secondly, the opponent may state that he does not recognise the authority in question; or thirdly, that though he has hitherto recognised it he does so no longer; and he may also employ two of these answers disjunctively. If, for example, a Conservative is pressed with some alleged statement of Lord Beaconsfield's, he can reply, Well, *if* Lord Beaconsfield

said that, I decline to follow him. The nature of an argument *ad ignorantiam* has not been very accurately explained by Logicians. It seems, however, to be an argument *ad verecundiam*, in which the reasoner holds himself out as the authority to which his auditor is bound to submit. "I tell you this or that, and you are bound to accept what I tell you," seems to be its import. Some writers, however, describe it as a fallacious argument addressed to persons who, from their ignorance, are unlikely to detect the fallacy. If among fallacies we include the assumption of unproved and unadmitted Premisses, this description does not differ very widely from the foregoing. The person who employs the argument assumes the truth of a Premiss of whose truth or falsehood he is aware that his hearers know nothing, and he expects them to receive it as true, solely because of his confident assertion (professedly grounded on his knowledge of the subject) that it is so. This is really an appeal to his own authority, adding to it perhaps a representation that he knows a proposition to be true when in point of fact he is well aware that the question is one about which there is no small amount of dispute among competent judges.*

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* Yet teachers must in many cases adopt this course. If it were necessary for them to stop and inform children at every stage that this or that item, whether of secular or religious knowledge, was disputed by some (on grounds which their hearers could not understand), they would convert the whole population of the country into Sceptics. Even the rotundity and revolution of the earth would be doubted.

Before leaving the subject of the Syllogism it may not be amiss to notice the Principles of Identity, Contradiction, and Excluded Middle which some writers of eminence have laid down as its ultimate laws. The Principle of Identity is generally expressed in the form Every B is B. Its practical application seems to be, that if anything is assumed to be B it must be steadily regarded as B throughout the whole of the following reasoning. The Principle of Contradiction is stated in the form No B is not B, or No B is non-B, and its practical application seems to be, that if anything is assumed to be B it cannot be regarded as non-B in any part of the following reasoning. The Principle of Excluded Middle, or Excluded Middle between Contradictories, is usually stated in the form Everything must be either B or non-B, or in some equivalent shape. On account, however, of the ambiguity of the words "either—or" (which may, or may not, be understood as asserting that the members of the Disjunctive are exclusive of each other), I prefer stating it, Whatever is not B is non-B. For if the proposition Everything is either B or non-B was understood to include the assertion that nothing could be both B and non-B, the Principle of Excluded Middle would include the Principle of Contradiction. Indeed it is evident that No B is non-B, and Nothing can be both B and non-B are merely two different ways of making the same assertion. The practical application of this third Principle seems to be that whatever is assumed not to be B must be steadily regarded as non-B throughout the following

reasoning. The truth of these three Principles cannot be questioned; nor is the fact that they are analytical propositions any objection to the doctrine that they form the basis of the science of Logic in the sense already explained, since the Dictum and the Axioms are also analytical propositions. In truth, as the Conclusion of a Syllogism adds nothing to what is asserted in the Premisses (although it usually does add something to what is asserted in either of them taken alone), the principles on which the science of Logic rests must be analytical propositions; and what the Logician has to discover (and to prove) is, what is or are the analytical proposition or propositions which express in the simplest way the general principles to which all right reasoning can be reduced. But when these three Principles are proposed as the basis of the science of Logic, they seem to me open to an objection of a different kind. They do not explain the office of the Middle Term in our reasonings as the Dictum and the Axioms do; and as the office of the Middle Term is the main thing to be explained, they consequently do not effect what they were designed to accomplish. If any Logician can deduce either the General or the Special Rules of the Syllogism from the Principles of Identity, of Contradiction, and of Excluded Middle, he will undoubtedly have proved that they may be substituted for the Dictum and the Axioms as the basis of the Syllogistic Theory. But so far as I am aware, no one has attempted any such deduction, nor do I see how it can be effected. The nearest approach that I have met with

is Dean Mansel's attempt to prove that illogical reasoning always involves a violation of one or other of these three Principles.* But in one of the cases which he considers—the mode AEE of the First Figure—he only succeeds in making this out by quantifying the predicate of the Major Premiss with the word "Some," and then taking this in the sense of "Some only." Nor would even this artifice enable him (so far as I can see) to extend his conclusions, for instance, to the mode III in any of the Figures. On this ground I decline to recognise the three Principles in question as constituting the basis of the Syllogism.

CHAPTER XVIII.

INDUCTION.

THE general problem of laying down rules for drawing or testing Inductions does not fall within the scope of the present work, but some remarks on the Logical character of the Inductive process may not be out of place. Using the word Induction* in its widest sense, so as to include what some writers call Analogy, it is an argument which concludes that because a certain number of Bs are Cs, Every B is C. This number may be either enumerated individually or in sub-classes, but most commonly the enumeration is an individual one. If, therefore, we put the

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* *Prolegomena Logica*, p. 268.

letters x, y, z, &c., for the individuals comprised in the enumeration, an Induction may be described as an argument which infers from the proposition x, y, z, &c. (all of which are assumed to be Bs), are Cs, that Every B is C.

Plainly such an argument is not logically conclusive, unless the individuals enumerated—the x, y, x, &c., with which we are dealing—constitute or rather are *alleged* to constitute *all* the Bs; for if there are any Bs about which no allegation has been made, it is possible, or at all events supposable, that these may differ in some of their properties from those which have been enumerated. I am of course assuming that Every B is C is a synthetical proposition, for, if it was an analytical one, its truth would be manifest without resorting to any Inductive proof. But if x, y, z, &c., constitute, and are alleged to constitute, the whole class B, the Inductive inference is conclusive, although the proposition Every B is C is synthetical. It is seldom possible, however, to form an Induction of this kind (which has been termed a Perfect Induction). Every B, means everything past, present, and future, which possesses the attributes connoted by the term B; but we seldom know, or find it possible to enumerate, all the present—still less all the past—members of the class B; while the future is entirely beyond our reach.*

* This observation is hardly applicable when we are drawing an inference from all the sub-classes to the principal class—from all the species to the genus. But in such cases the proof that all the members of any given sub-class of Bs are Cs, is usually Inductive and involves the difficulties mentioned in the text.

A Perfect Induction is therefore possible only when the connotation of the term B is such as to render future members of the class B impossible, while the past and present members of it are not very numerous, and are capable of being certainly ascertained. There are a few such classes; and universal propositions in respect of them are capable of being proved by a Perfect Induction. If I asserted that All the members of the Irish Parliament which passed the Act of Union are dead, I might perhaps be able to establish this proposition by examining a list of the members of that Parliament, and proving the death of each of them individually, though it must be confessed that this would be a very troublesome and difficult task. If I asserted that certain peculiarities were to be found in all Sir Walter Scott's novels, I might prove this by taking each of his novels in detail, and showing that every one of them exhibited these peculiarities: and I might in the same way establish some general characteristic of all Tennyson's poems that have hitherto been published: but as Tennyson may live to publish other poems whose characteristics will be different, I could not prove any proposition concerning *all* Tennyson's poems by a Perfect Induction. Mr. Mill regards such propositions as these not as *general* propositions at all, but as mere abbreviations of a definite number of *singular* propositions—expressed as it were in a kind of shorthand. All Scott's novels possess the attribute C, would in his view be merely a short way of writing, Waverley possesses the attribute C, and so does Guy

Mannering, and so does Ivanhoe, proceeding with the enumeration until we get to the end of the list. And if this view were correct, no general proposition could be proved by a Perfect Induction, and that kind of argument might be wholly discarded in a treatise on Logic, or at most dealt with as a kind of immediate inference from singular propositions, almost unworthy of the name of reasoning. But I cannot regard the propositions we have been discussing as mere abbreviations of a definite number of singular propositions. A person might be acquainted with all Sir Walter Scott's novels without knowing that all of them proceeded from his pen, or he might be acquainted with the novels and know that they were Scott's without being aware that Scott had written no others. It is even possible that all the singular propositions which enter into the Induction might be true, and the Inductive Conclusion false; as would be the case if it should be proved hereafter that Scott wrote another novel, not hitherto known as his, and unlike his other productions. No doubt if this proved to be the case, the Induction could hardly be regarded any longer as a Perfect Induction; but other instances might be mentioned in which it is doubtful whether the alleged Perfect Induction is really perfect, though the truth of all the singular propositions upon which it is grounded is conceded. It therefore appears to me that Perfect Inductions are sometimes possible, and that when possible they are not merely abbreviated expressions for a definite number of singular

propositions, but general propositions—propositions which assert some predicate of everything that possesses a given attribute or given attributes. But the instances in which such conclusions can be drawn are very few, and for the most part very unimportant. I can no doubt always make an Inductive argument conclusive, by *asserting* that the individuals contained in my enumeration constitute the whole class to which they belong: but though this assertion would make the argument logically conclusive, it would be so evidently false or doubtful in most instances that nobody would think of making it. Practically men do not assert that the individuals enumerated in an Inductive argument constitute a whole class, unless the allegation is at least plausible; and it is not often that it possesses even plausibility. No Logician could of course admit such assertions as These instances include *nearly* all the Bs, or *x*, *y*, *z*, &c., are *as good as* all the Bs, as rendering the Induction Perfect or the reasoning conclusive. What is true of nearly all, may not be true of all; and as to the phrase "as good as all," it either means nearly all, or else that the instances enumerated are "as good as all" for the purposes of reasoning. But they are not as good as all for the purposes of reasoning, unless everything that is true of them is true of all; and if this is intended to be implied, the Logician should insist on its being explicitly stated. Vague, loose, indefinite and elliptical expressions allowed to pass unchallenged are among the most fertile sources

of fallacy and bad reasoning. The Induction is only Perfect if the individuals enumerated are *literally* all the Bs. Whenever the assertion falls short of this, the Induction is Imperfect. In the vast majority of Inductions, however, including the most important of them, it is admitted that x, y, z, &c., do *not* constitute all the Bs. Here it is evident that the reasoning is inconclusive until it is asserted that whatever is true of x, y, z, &c., is true of all the rest of the Bs. This assertion can seldom be more than probable, and if the several singular propositions (x is C, y is C, z is C, &c.) are all certain, the probability of the conclusion will be exactly the same as that of the proposition that whatever is true of x, y, z, &c., is true of all the Bs: and no doubt if there are any instances in which this last proposition is certainly true (the several singular propositions being likewise certain), the conclusion must be accepted as certainly true also. Usually, however, we have only probability: and the probability that what is true of x, y, z, &c., is true of all the Bs (or of all the rest of the Bs), will vary with the number and nature of the individual cases alleged, and also with the subject-matter we are dealing with. This probability may be so high as to be almost undistinguishable from certainty, or it may be so low as to raise but a very slight presumption. It would be impossible here to discuss all its variations, and I must therefore content myself with pointing out the assumption involved in every Inductive argument which does not amount to a Perfect Induction—the assumption which is requi-

site to render the reasoning in question logically conclusive.

The reasoning in a Perfect Induction then appears to be as follows—

> *x*, *y*, *z*, &c., are Cs,
> *x*, *y*, *z*, &c., are all the Bs,
> ∴ All Bs are Cs,

while in an Imperfect Induction it is as follows—

> Every attribute of *x*, *y*, *z*, &c., is an attribute of all the Bs,
> C is an attribute of *x*, *y*, *z*, &c.,
> ∴ C is an attribute of all the Bs,

which last proposition is evidently equivalent to All Bs are Cs. It will be seen that the Imperfect Induction thus expressed is quite as conclusive as the Perfect, and though one of its premisses is usually (if not always) open to some doubt, the same thing sometimes happens in the case of a Perfect Induction. The real distinction between them is that in the latter all the members of the class are alleged to have been enumerated, while in the former it is confessed that the enumeration is not complete.

There is no special difficulty in reducing an Imperfect Induction to our Syllogistic Forms, but a Perfect Induction is not so easily brought under them. It in fact appears to be a Syllogism of the Form AUA, and the Hamiltonians accordingly contend that we have here not merely an argument in general use into which the Proposition U enters, but one which cannot

be adequately expressed in any other way. To this it might perhaps be sufficient to reply that the Proposition U in this as well as other instances is a complex or compound proposition, and that the entire reasoning therefore embraces at least two Syllogisms. In fact if the proposition x, y, z, &c., are all the Bs, was only true of the Bs taken collectively, the conclusion would not follow. A predicate might be true of every individual ship's crew, for example, and all sailors might be (collectively) identical with all ships' crews, and yet the predicate might not apply to even a single sailor. In order to warrant the conclusion drawn from it in a Perfect Induction, the proposition x, y, z, &c., are all Bs, must therefore be understood to assert that x, y, z, &c., are each of them Bs as well as that when taken together they constitute the whole class of Bs: and as these two assertions are evidently distinct from each other, and one might be true while the other was false, it would seem desirable to state them separately in Logic. However, there are some peculiarities of singular propositions (including propositions whose predicates are singular terms) which it here becomes desirable to mention. It has been observed that "Some" is altogether inapplicable as a note of quantity in the case of a singular term. This observation is equally true of "All," which in Logic means "each and every," and therefore implies that the term to which it is affixed is not a singular term. It has been stated that singular propositions—propositions whose subjects are singular terms—may be treated as uni-

versals, because their denotations consisting of a single individual cannot be sub-divided, and must be taken entire: but while the proposition John Smith is an Englishman may thus be usually treated as A (and can never be treated as I), it is not really A. The Propositional Form A requires the *All* or *Every* to be expressed, and in the singular proposition it is not only unexpressed, but would be altogether out of place if inserted. Singular terms occur more rarely as predicates than as subjects of propositions, but such predicates are occasionally found also. And here also we find it necessary to modify the ordinary Logical rules. The rule that the predicate of an affirmative proposition is particular, while that of negative proposition is universal, supposes that the predicate is not a singular term. A singular term can be neither universal nor particular, though for certain purposes it may be treated as universal. Properly speaking, such propositions as Hyde was Clarendon, or Tully was Cicero, are not, nor are their predicates, either universal or particular; nor if treated as universals can we regard them as either A or U. So in the proposition All the Bs are x, y, z, &c. (x, y, z, &c., being the predicate), the singular term which forms the predicate is neither universal nor particular, and the proposition itself cannot be properly described either as U or as A. And it seems to me that the proposition x, y, z, &c., are all the Bs, is merely this last proposition with its predicate written before the subject, and that therefore, while it must be confessed that it is not A, neither is it

U. As expressed, it has the fault of apparently conveying only a collective assertion, though it is intended to be applied distributively in the conclusion: but this defect could be remedied by stating it in the form Every B is either x, or y, or z, or, &c.—a form which seems to me to bring out its real meaning most accurately. A Perfect Induction from this point of view should be written

x, and y, and z, and, &c., are Cs.

Every B is either x, or y, or z, or, &c.,

∴ Every B is C.

If x, y, z, &c., are sub-classes, not individuals, the advantages of this form are obvious. On the whole, it does not appear to me that a Perfect Induction eludes our Syllogistic rules (the peculiarities of propositions involving singular terms being borne in mind), or that it affords any reason for adding the Hamiltonian U to the Propositional Forms commonly adopted by Logicians. And I may add that when a Perfect Induction is stated in the above form, and x, y, z, &c., are sub-classes, not individuals, the Minor premiss is often an unmistakeable A. If, for example, this premiss was Every man is either white, black, brown or red, it is plain that the predicate should be quantified by *Some* not *All*: for there are white things, black things, brown things, and red things which are not men. This observation also disposes of the assertion that every disjunctive proposition of the form

Every B is either C or D or E or F, is reducible to U. Such a proposition by no means implies that all the Cs, all the Ds, all the Es, and all the Fs are Bs, and consequently its disjunctive predicate is not universal but particular. And so of the Disjunctive proposition in its most general form. Either B is C or D is F, reduces to Every case-of-B-not-being C, is a case of-D-being-F, which does not imply that Every-case-of D-being-F is a case-of-B-not-being-C. Its predicate is thus particular. And whenever the predicates of the kind of propositions which we have been considering are not particular, it is only because they consist of singular terms which do not admit of particular quantification.

The distinction between the classes respecting which Perfect Inductions may possibly be made, and those with respect to which a Perfect Induction is impossible, coincides pretty nearly with that sometimes drawn between Definite and Indefinite Classes. But it must be recollected in the first place that every class-name connotes one or more attributes which determine its denotation, and in the next that the number of individuals comprised in any class at a given moment is definite, though in many cases no one possesses the knowledge required to enumerate them all. The important point, therefore, is not that the number of individuals now or hitherto comprised in the class is definite and known, but that this number is completed so that no additions can be made to it in the future. It is this circumstance that renders it at least conceivable that all

the individual members of the class may be enumerated, and that some attribute may thus be proved to belong to the class by means of a Perfect Induction.

CHAPTER XIX.

FALLACIES, AND REMARKS ON SOME WELL-KNOWN SOPHISMS.

THE division of Fallacies usually adopted by Logicians was into Fallacies *In Dictione* and Fallacies *Extra Dictionem*. A Fallacy *In Dictione* is an argument whose defect (or rather the *concealment* of whose defect) arises from the language employed; while in Fallacies *Extra Dictionem* the fault lies in the train of thought itself, independently of the language in which it is expressed. When a Fallacy *In Dictione* is translated into some other language, its weakness usually becomes immediately apparent, whereas a Fallacy *Extra Dictionem* is for the most part as difficult to detect and to refute when expressed in English as in Latin. Occasionally indeed the same ambiguities or deficiencies are found in several languages (especially if these have a common origin), and in such cases translation may fail to bring out the weak points of a Fallacy *In Dictione*. Fallacies *In Dictione* thus correspond very nearly to Fallacies of Ambiguity, that is, arguments in the course of which the same term is used in diffe-

rent senses (for it is no fault in an argument that one of the terms employed is susceptible of two meanings, if the reasoner adheres to one of these only). Fallacies *Extra Dictionem*, on the other hand, include all invalid Syllogisms and all longer reasonings which, on being reduced to the Syllogistic Form, are found to include one or more invalid Syllogisms—thus corresponding to what Archbishop Whately calls Logical Fallacies. But if an argument is free from ambiguity in its terms, and its Conclusion follows from its Premisses, it is not easy to see why it should be called a Fallacy. The Conclusion of any Syllogism may be doubtful or false if one or both of its Premisses be so; but no writer on Logic calls every Syllogism with a doubtful or false Premiss a Fallacy. The term Fallacy is generally reserved for a Syllogism with a doubtful or false Premiss, which for some reason or other is likely to be accepted as true by the ordinary reader. This description, however, really carries us back to a prior Syllogism which labours under some other defect than that of having a doubtful or false Premiss. The ordinary reader has some reason (or reasons) for accepting the Premiss in question as true, but this reason is insufficient since the Premiss is in reality doubtful or false. Let us then *state* this insufficient reason, and we shall obtain a previous Syllogism whose object is to prove a Premiss of the subsequent one—a Syllogism which must be invalid, since the reasons assigned for drawing its Conclusion are insufficient. It is to this previous Syllogism, and not to the subsequent one, that the term

Fallacy seems to be properly applicable; but when a train of reasoning includes more than one Syllogism there is always another kind of Fallacy to be guarded against—namely, passing from one meaning of an ambiguous term (or of an ambiguous proposition) to another, when we proceed to employ the Conclusion of one Syllogism as a Premiss in a subsequent one. This is of course a Fallacy *In Dictione;* but instead of being a defect in either of the Syllogisms separately, it here slips in between the two.

Writers on Logic have usually endeavoured to clear up a number of famous Fallacies or Sophisms; and though the task does not strictly speaking belong to the Science, the exercise is no doubt an useful one. I therefore conclude this Treatise by examining a few of the most celebrated Fallacies of this kind.

Protagoras undertook to teach Euathlus how to plead causes in Court for a reward which was to be paid when Euathlus won his first cause. Euathlus, after receiving the instruction, did not choose to become a pleader, and Protagoras sued him for the amount agreed upon. Euathlus appeared before the Judges in his own defence. Protagoras proceeded to argue as follows:—If you win the cause, you are bound to pay me by our agreement; but if I win it, you are bound to pay by the decision of the Court; in either case, therefore, you must pay. Euathlus replied, If I win the cause, I am not to pay by the decision of the Court; if I lose it I will not have won a cause, and therefore I am not to pay by our agreement. In either case, therefore, I am not to pay.

Here, if the Court regarded the spirit rather than the letter of the contract, there can be no doubt that the decision would have been in favour of Protagoras. He undertook to render Euathlus a competent pleader, and was to be paid for doing so. The winning of a cause was merely a test of the proficiency acquired by the pupil. The terms had clearly been arranged on the assumption that Euathlus *would* plead causes, and he was not at liberty to deprive Protagoras of the reward he had earned, merely because he changed his mind, and preferred adopting some other profession after having acquired a capacity for pleading at the cost of his teacher's time and labour. But, on the other hand, if the letter of the contract alone was to be regarded (and both reasoners seem to have relied solely on the letter), it is equally clear that Euathlus was entitled to judgment. When Protagoras commenced his action Euathlus had not won any cause; and even when the suit came on for hearing he was still in the same predicament. The judge should simply have asked Protagoras, What cause do you say Euathlus has won? The only answer Protagoras could give would have been, This cause. To this the judge would answer, This cause is still undecided. You have brought Euathlus here, alleging that he has won a cause and consequently owes you this money under the contract. If you cannot tell me what cause he has won, I must dismiss the case. But another question would then arise. Euathlus had *now* won a cause. Could not Protagoras commence a *new* action, relying

on Euathlus's victory in the *former* one as entitling him to his reward under the contract? And to this question the answer must be in the affirmative, if the terms "winning a cause" were held to be wide enough to include a successful resistance to an action brought against Euathlus himself. But in any event Protagoras's right of action could not arise until the Court had decided in favour of Euathlus.

The Fallacy of the Crocodile is of a somewhat similar character. A Crocodile is supposed to have taken a child, and then tells the mother that if she will *truly* inform him whether he will determine to eat it or not, he will give it up to her. On her giving either answer, an argument arises similar to that which has just been considered. If she says You will resolve to eat it, the Crocodile replies that, to make this answer true, he must eat the child: if she says You will resolve not to eat it, he replies that the answer is not true, and, therefore, he will eat it. The mother, of course, can have recourse to a counter-argument of a similar character. In solving this Fallacy we must commence by ascertaining the meaning of the Crocodile's promise. Did the Crocodile mean, If you tell me truly whether I will *resolve* to eat the child or not, I will release it; or, If you tell me truly whether I will *in fact* eat the child or not, I will release it? If the former, his resolution is plainly not regarded as unalterable. Assuming, then, in the first place, that he resolves to eat the child, and that the mother truly states his intention, he is bound by his promise to change his mind and deliver up the child to the mother.

In the second place, if he resolves to eat the child, and the mother untruly states that this is not his intention, there is no reason why he should not proceed to devour it. In the third place, if he resolves not to eat it, and the mother truly states this intention, he is of course bound to deliver it up; while, lastly, if he resolves not to eat it, and the mother falsely states that he does, he is not bound to deliver it up, though he has no reason to change his intention of not eating it. Supposing, however, that the Crocodile meant to say, If you will truly tell me which of the two events will *in fact* happen, I will release the child, he has made a promise which he cannot possibly fulfil, unless the event about to occur is that he will not eat the child. For, if the event about to occur is that he will eat the child (compelled to do so, we will suppose, by Fate or by some uncontrollable impulse), he cannot possibly fulfil his promise of releasing it in case the mother truly predicts that event. In such a case she could, of course, prove by a Syllogism that he had broken his promise to her, which would be a very natural consequence of making a promise which he could not keep. The fact is, however, that a promise of any kind supposes that the event is in some way or other contingent on the will of the person who makes it; and, on the contrary supposition, the promise itself becomes absurd, and for that reason leads to absurd consequences. If Fate had irrevocably determined either that the child should or should not be eaten, neither the Crocodile's promise nor

the mother's prediction (whether true or false) could have any influence on the event.*

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* Several of the Fallacies instanced by the ancients turn on the supposition of such an irrevocable Fate, which I do not admit; but the error in such cases (as in the above instance) arises from introducing the idea of human agency *as something contingent* into the reasoning at a later stage—a supposition which is inconsistent with the former. Such is the argument ascribed to the sick man; If I am fated to recover, I shall recover whether I employ a Physician or not; if I am fated to die, I shall die whether I employ a Physician or not; consequently the employment of a Physician can have no effect on the issue, and I will not employ him. This reasoning is inconsistent with the assumption of an universal irrevocable Fate, since it assumes that the employment or non-employment of a Physician is in the power of the sick man. If *all* things are determined by an irrevocable Fate, this Fate has already determined either that a Physician shall be consulted or that he shall not, and all deliberation on the subject must, therefore, be resultless. And while this consideration disposes of the *practical* Conclusion, I will not call in a Physician, it may be added that even the *theoretical* Conclusion, It is indifferent whether a Physician is called in or not, does not follow from the Premisses. For Fate does not exclude second causes, unless we assume that Fate never makes use of one thing to accomplish another. Fate may have not only ordained that the sick man shall recover, but that his recovery shall be caused by the attendance of a Physician—this attendance being, of course, ordained by Fate also. Nor could the sick man even infer, It is indifferent whether *I resolve* to call in a Physician or not: for Fate may have ordained that the sick man's resolution to call in the Physician (a resolution likewise produced by Fate) shall be the cause of his attendance, and that his attendance shall be the cause of the ultimate recovery which has been ordained by Fate. All that can be inferred from such Premisses is, that if an event is determined by an irrevocable Fate *independently of second causes*, such causes can have no influence on it: while, if an event is determined by irrevocable Fate (second causes *not* being excluded), this event cannot be affected by any agency *which is independent of this irrevocable Fate*—two propositions which no one will feel disposed to dispute.

The Fallacy of the Liar is of this kind. A man says I lie. In this assertion either he is telling the truth or he is not. But if he tells the truth he is lying (since that is the statement which by hypothesis is true); and if he is telling a lie, he tells the truth, inasmuch as he has *said* that he lies. Here again two cases are possible. Either the man has made some previous statement to which he refers in the words "I lie," or he has not. In the former case he has made two contradictory statements —one of them a proposition which we may describe as B is C, and the other, "I lie," *i. e.* the proposition B is C is false. Of these two contradictory assertions one is true and the other false, and the speaker has not told simple truth or simple falsehood, but has made a mixed statement, part of which is true, while the other part is false. The correct mode of expressing the second of these statements, however, is not *I lie*, but *I have lied* (or *I am going to tell a lie*, if the order of the two assertions is inverted). Next, suppose that the words "I lie" do not refer to any previous or subsequent proposition, but stand alone. Now, *I lie* is merely a short way of saying *I assert a false proposition*, which, in the supposed instance, is not the case, inasmuch as the speaker has not asserted *any* proposition. But, though the statement *I lie* is thus false, we must not from thence conclude that the speaker has spoken truly: for *I speak truth* is equivalent to *I assert a true proposition*, whereas the speaker has not asserted any proposition whether true or false. "I lie" and "I speak truly" do not include all possible contingencies, for there is a third

possibility, namely, *I assert nothing.* If, however, an attempt is made to treat the words *I lie* as themselves constituting the proposition about which the assertion *I lie* is made, we really turn this single assertion into two inconsistent propositions, and so obtain a complex statement, partly true and partly false, as before. The full expression for what is intended would indeed be "I lie—and this assertion 'I lie' is itself a lie."

The Liar has been carried somewhat further in a Fallacy founded on the saying of Epimenides the Cretan, The Cretans are always liars. Now, if Epimenides meant to assert (which most assuredly he did not), that no Cretan ever spoke the truth on any occasion whatever, this assertion would imply that Epimenides himself never spoke the truth, and, therefore, was not speaking truly in the present instance. Such a statement being inconsistent with itself, naturally leads to inconsistent consequences. But the manner in which the argument has been worked out involves further fallacies. It proceeds thus:—(Epimenides says) All the Cretans are liars. But Epimenides is a Cretan: therefore he is a liar. (It will be seen that the argument rather takes the shape of an argument *ad hominem*, addressed to Epimenides, than an ordinary train of reasoning.) We now commence another Syllogism. That all the Cretans are liars is an assertion of Epimenides: Epimenides is a liar: therefore this assertion is false. Pausing here, it will be seen that this Conclusion only follows if Epimenides is a liar means *All* Epimenides's assertions are false. We now go on. It is false that (all) the Cretans

are liars: therefore the Contradictory of this false assertion is true: consequently they are truth-tellers. At this stage a double fallacy is introduced:—First, the Contradictory of the (we will suppose false) proposition that (All) the Cretans are liars is *Some* of the Cretans are *not* liars; whereas the Conclusion means that *None* of them are so, or that all of them are truth-tellers: and, secondly, if by Liars we mean (as has been assumed in the whole argument) persons who tell *nothing but lies*, then the Not-liars cannot be identified with Truth-tellers, unless by the latter term we mean persons who *sometimes* tell the truth. The argument now proceeds: All the Cretans are truth-tellers. Epimenides is a Cretan, therefore he is a truth-teller. But that All the Cretans are Liars is an assertion of Epimenides: he is a truth-teller: consequently this assertion is true: therefore All the Cretans *are* Liars—at which point we have completed the circle, and can, if we choose, make a fresh start and go round it a second and a third time. It is evident that in the last branch of this reasoning, truth-teller is taken to mean a person who tells nothing but truth: and the whole reasoning proceeds on the assumption that there are only two possible cases, viz., that *All* the Cretans *on every occasion* state what is false, and that *All* the Cretans *on every occasion* state what is true. But any general assertion, that no member of a class *to which the speaker belongs* ever speaks truth, must be false, since the supposition of its truth is self-destructive. It supposes that *all* assertions made by a person belonging to a particular class are false, and yet that *one* of these

very assertions—this one—is true: and a fallacy is sufficiently refuted for all practical purposes, if we prove that it assumes a Premiss which cannot possibly be true.

Another favourite fallacy was the following:—If a body moves (or as it is sometimes put, *begins* to move), it must move either in the place where it is or the place where it is not. But if it moves in the place where it is, it remains in the place where it is; consequently it does not *change* its place, and therefore it does not move. If, on the contrary, it moves in the place it is not, then it must be in the place where it is not, which is absurd. Dean Mansel attempts to solve this fallacy by saying that it moves partly in the place where it is, and partly in the place where it is not. If this means that one part of the moving body is in the place where it is, and another part of it is in the place where it is not, one branch of the above dilemma applies to the former part, and the other branch to the latter; while if it is meant that each atom or particle of the moving body is partly in the place where it is, and partly in the place where it is not, this view is by no means free from difficulty. In attempting a different solution I must begin by calling attention to an ambiguity in the word Place. The Place of a body may either mean the precise space which it occupies at a given moment (supposing it at rest), or it may include a limited portion of the surrounding space. In the latter meaning there is no difficulty in supposing a body to move in the place where it is, for in that sense "the Place where it is" is large enough

to admit of the body moving in it; and as in moving it gets nearer to one end of this limited space, we begin to drop out some of the surrounding space towards the other end, and to take in more of the surrounding space in this direction when we speak of the Place of the moving body, or of the Place where it is. But if by the Place of the body, or the Place where it is, we mean the precise space which it actually occupies at a given instant, no doubt the body as a whole cannot move in the Place where it is; and if we reduce the body to a single atom, no motion (except, perhaps, revolution on an axis) would be possible in the Place where it is. The answer to the fallacy, however, becomes clear as soon as we fix in our minds the idea or notion of Motion. Motion is a change of Place occurring in Time—the Place of the moving particle at any one instant of Time being different from its Place in the immediately preceding instant. Time is thus necessarily involved in the very notion of Motion. Now, as soon as we suppose the smallest portion of Time—the millionth part of a second—to have elapsed, "the Place where it *is*" has become "the Place where it *was;*" and the Place where it *is* not has become the Place where it *was* not; and whether the particle is *still* in the Place where it *was* an instant ago, or has moved to the Place where it *was not* at that time, is to be determined by experience, not reasoning. If the particle is at rest, it is still in the Place where it was; but not so if it is in motion. In that case the Place where it is *now* is different from what was (correctly)

described as "the Place where it is," only one-millionth part of a second ago. What the argument really proves is, that a body cannot change its Place in *an indivisible instant*—that we cannot suppose any change in its Place unless we allow a sufficient Time to have elapsed to enable "the Place where it *is*" to have become "the Place where it *was*." And this I think the argument does prove. But if we suppose a moving material particle to have changed its place without any lapse of Time, it will be easy to prove, mathematically, that its velocity is infinite. The argument, therefore, may be considered as proving that a material particle cannot move with an infinite velocity. But if the velocity be finite, the change of Place and the lapse of Time always go hand in hand, and neither can be separated from the other. The possibility of such motion is wholly unaffected by the argument.

The fallacy of Achilles and the Tortoise is of this kind. Let Achilles run a race against a Tortoise, giving the latter the start, by, say a mile. Then, although Achilles runs ten times as fast as the Tortoise, he can never overtake it; for while he is traversing the mile (the Tortoise's original start), the Tortoise will have run a tenth of a mile; while Achilles is running this tenth of a mile, the Tortoise will have run a hundredth part of a mile; while he is running this hundredth part, the Tortoise will have run the thousandth part, and so on for ever. Therefore Achilles will never overtake the Tortoise. This fallacy has been thus solved by Dean Mansel. Let the time

which Achilles takes to run a mile be a; then (as he is supposed to continue to run at the same rate) the time required to run $\frac{1}{10}$ of a mile will be $\frac{a}{10}$; the time required to run $\frac{1}{100}$ of a mile $\frac{a}{100}$, and so on. What is really proved then is, that Achilles will not overtake the Tortoise during the time represented by the Series

$$a + \frac{a}{10} + \frac{a}{100} + \frac{a}{1000} + \text{\&c. } ad\ infinitum.$$

The fallacy consists in assuming that because this series contains an infinite *number of terms* (or at least may be continued *ad infinitum*), its *Sum* (or as some Mathematicians prefer to say, the limit of its Sum) is therefore infinite. But every Mathematician knows that a Series may contain an infinite number of terms, and its Sum may, notwithstanding, be finite; and the Sum of this particular Series is easily shown to be $a + \frac{a}{9}$—a fact which indeed anyone who understands Decimal Fractions can see—for the Series is in fact a multiplied by $1 \cdot 1'$, this Circulating Decimal being equivalent to the vulgar fraction $\frac{1}{9}$.

Enough perhaps has been said of Fallacies turning on the assumption of an irrevocable Fate. The following instance from Archbishop Whately's collection may, however, be noted :—

He who necessarily goes or stays, is not a free agent.
You must necessarily go or stay;
∴ You are not a free agent.

Here the Major Premiss means He who necessarily goes, or who *necessarily* stays is not a free agent, though to avoid prolixity the word "necessarily" is

expressed but once. In fact the best way of stating the Major Premiss would be Neither he who necessarily goes, nor he who necessarily stays is a free agent.* The Minor Premiss, on the other hand, means You must necessarily go-or-stay. It is impossible to construct a valid Syllogism from such Premisses, for as soon as the second "necessarily" is inserted in the Major Premiss, and omitted in the Minor Premiss (the meaning of which would be altered by inserting it), we find that we have no longer a Middle Term common to both.

It sometimes happens that we can either treat an argument as a valid Syllogism with a true Conclusion, or as a formal Fallacy, according to the sense in which the Conclusion is understood. Thus—

> Socrates is different from Coriscus;
> Coriscus is a man;
> ∴ Socrates is different from a man.

Socrates is certainly different from *a* man—namely, from Coriscus—but we cannot thence infer that he is different from *any other* man. If, therefore, the Conclusion is taken to mean, that Socrates is different from *some one particular man*, it is undoubtedly true and follows

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* A Proposition whose parts are united by the particles "Neither—nor" is not a Disjunctive Proposition. It is a compound Proposition composed of two negative Categoricals. These in the above instance are, 1st, He who necessarily goes is not a free agent, *and* 2nd, He who necessarily stays is not a free agent. The two are thus connected by the particle *and*, instead of *or*.

from the Premisses; but if the meaning be, that Socrates is different from *any* man, the Syllogism is invalid for what is in substance an Illicit Process of the Major Term—as will be seen by substituting the simpler word "*not*" for "different from." And in fact in Negative Syllogisms the fault which has been called Illicit Process of the Major Term will always disappear if we understand the Predicate of the Conclusion as Singular instead of Universal—that is, if we understand No B is *a* C, to mean No B is identical with some individual C which is present to our thoughts at the time.* The Universal predicate, if intended, would be better expressed (after Hamilton) by No B is *any* C; and if this mode of expression was adopted, No B is *a* C would always imply a singular predicate—being in fact equivalent to No B is *this* C. But we do not require to introduce any such form as this last; for *No B is this C* is better expressed by *This C is not B.*

The following argument has occasioned some perplexity to Logicians:—

A good Pastor is prepared to lay down his life for the sheep;
Few Pastors of the present day are so prepared;
∴ Few Pastors of the present day are good Pastors.

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* An individual C, present in our thoughts as an individual, must be regarded as singular rather than particular, even though it may not be sufficiently described to distinguish it from all other Cs. This meaning of No B is *a* C, therefore, does not reduce it to Hamilton's No B is *some* C, which, with him, means No Bs are some Cs.

Here the Major Premiss is intended to mean, that *all* good Pastors are so prepared, and should be so expressed; while if it is intended to imply also (which, however, is not essential to the argument) that *all* who are so prepared are good Pastors, this should likewise be stated in terms. But when we come to deal with the Minor Premiss a further difficulty arises. *Few* is not a term of quantification admitted in ordinary Logic; though if it means *a few* it may be dealt with exactly like the *Some* of Logic, and is subject to similar rules. But *Few* Bs are Cs sometimes means Few, *if any*, Bs are Cs, and therefore does not imply Some Bs are Cs, but that *either* Some (only) *or none* are so. Taking it in this latter sense, the above reasoning may be exhibited as follows:—

All the Pastors, each of whom is a good Pastor, are Pastors each of whom is prepared to lay down his life for the sheep.
(All) the Pastors (if any—of the present day), each of whom is prepared to lay down his life for the sheep, are few in number.
∴ (All) the Pastors (if any—of the present day), each of whom is a good Pastor, are few in number.

I have adopted this prolix method of stating the argument to avoid an objection that if Few Bs are Cs should be treated in Logic as The Bs-which-are-Cs are few (Bs-which-are-Cs being the subject of the proposition, and few the predicate), this predicate applies to the subject collectively, not individually, and the above

proposition cannot therefore be combined in a Syllogism with another proposition, in which some other predicate is applied to the Bs-which-are-Cs individually or distributively, without falling into a Fallacy of Composition or Division. But when what is collective and what is distributive in each proposition is expressed as above, this objection falls to the ground, and the argument (provided I have correctly expressed what it was intended to convey) can be shown to be valid.* Not that the Syllogism given above is a valid Syllogism, but that the reasoning becomes valid on supplying another step (and consequently another Syllogism) which is necessary to complete it. It has not been asserted that All the Pastors who are prepared to lay down their lives for the sheep are good Pastors, and we must assume the contrary to be possible at least in some instances. But the *only* Pastors of the present day who *can* be good Pastors are those who are prepared to lay down their lives for the sheep;

* Archbishop Whately, besides rendering Few Bs are Cs by The Bs-which-are-Cs are few, as above, gives Most Bs are not Cs as another rendering. But the fact is that propositions in which the term *most* occurs in connexion with the subject present the same Logical difficulties as those where the corresponding term is few; for "most" as well as "few" applies to the whole collection rather than to its individual members. The Port Royal Logic treats such a proposition as equivalent to Many Bs are not Cs. This, however, is erroneous. If the Bs are a numerous class, many Bs may be Cs, and many Bs may not be Cs at the same time. Many Bs are not Cs, is not, therefore, the equivalent of Few Bs are Cs. And if "many" is treated as a predicate, it involves the same Logical difficulties (as to its collective or distributive use) as "few."

and if these are few in number, the only Pastors of the present day who *can possibly* fall into the class of good Pastors are few in number. If *all* of them fall into that class their number is few; if *some only*, it is fewer still; while if *none* do so, we have seen that *Few* Bs are Cs means, Few *if any* Bs are Cs, and so covers even this extreme case. But the reasoning is not logically complete until we have stated that any part of *few* is likewise *few*, and that the term Few does not exclude *none*. If, however, the Conclusion is taken to mean that some of the Pastors of the present day *are* good Pastors (although these are few in number), the reasoning is invalid. Treated as a single Syllogism, it has an Undistributed Middle, and this defect is not remedied by expressing at length the considerations above adverted to. But it would become a valid Syllogism if we understood the Major Premiss as asserting that All Pastors who are prepared to lay down their lives for the sheep are good Pastors; or to adopt our former mode of expression—

> All the Pastors, each of whom is prepared to lay down his life for the sheep, are Pastors each of whom is a good Pastor;

on supplying to which Major Premiss our former Minor Premiss, the Conclusion will be found to follow. Indeed three-fourths of the difficulties which we meet with in Logic arise from inaccuracies of expression which leave us in doubt as to what was intended to be asserted; and Fallacies of all kinds usually disappear

as soon as the several propositions employed are thrown into such a form that all traces of ambiguity in their meaning is removed. Whenever, therefore, a fallacy is suspected, we should take especial pains to set out the various propositions employed in the fullest and most accurate manner in which they can be expressed. When this is done, the fault (if there be one) usually becomes visible.

Some writers include erroneous Inductions among fallacies, but I think improperly. An Induction, as we have seen, is seldom if ever *perfectly* conclusive. Room must be left for possible exceptions, however strong may be our confidence that there are none in fact. It is always conceivable that the Premisses of an Induction may be true while its conclusion is false.* And different Inductions are probable in various degrees from the highest to the lowest, so that it becomes impossible to draw an absolute line of demarcation between the good and the bad. Moreover, if we throw an Induction, as commonly expressed, into Logical form—in which case it will not form a valid Syllogism, for its Conclusion will contain more

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* Meaning by the Premisses the individual instances relied on. But if we consider the *whole* of these instances as forming *one* Premiss of the Induction, and the other Premiss as If any property belongs to This B, This B, This B, and This B (enumerating them) it belongs to all the Bs, the Syllogism is valid, but the Hypothetical Premiss is doubtful and its doubtfulness admits of so many degrees that no absolute line of demarcation can be drawn between a good and a bad Induction. I assume, of course, that the Hypothetical Premiss is not known to be false. That would make the Induction undoubtedly bad; but only in the same sense that any other Syllogism with a Premiss known to be false is a bad Syllogism.

than the Premisses—by substituting letters of the Alphabet for the terms, a good Induction and a bad Induction will be precisely alike, notwithstanding that we have taken all possible pains to state them with accuracy. Each will run pretty much as follows:—

> This B, and This B, and This B (enumerating them), are Cs;
>
> ∴ All Bs are Cs.

This Conclusion evidently cannot be drawn if any of the Bs are known *not* to be Cs; but where none are known not to be Cs, and several are known to be Cs, the Conclusion will always be more or less probable. The degree of probability depends as much on the nature of the subject-matter as on the number of instances enumerated; while the nature of the subject-matter in each instance belongs to the Science which deals with that subject-matter, and not to Logic. Thus, to take an example from Mr. Mill, it is Chemistry, not Logic, that tells us that greater uniformity is to be expected in the case of Simple Substances than in that of very composite bodies, and it is to the same Science that we must refer if we wish to know what bodies are simple, and what bodies are composite. And when we have acquired this knowledge from Chemistry, our reasons for placing greater reliance on Inductions relating to Simple Substances (if such there be) can be expressed in the Syllogistic Form.

Circumstantial evidence in individual cases is extremely like an Induction in its characteristics. In the case of a murder, for example, the whole stress

of the circumstantial evidence adduced lies in proving that *all* the circumstances adduced could not be combined in any person except the murderer. He was near the spot. That may prove little, for perhaps twenty other persons were near also (though if it were at night in a lonely locality the chance would be considerably augmented.) He had a quarrel with the deceased man. So perhaps had a dozen others. He had threatened his life. So perhaps had others. He had a weapon similar to that used on the occasion. Possibly such weapons were common in the neighbourhood; and so on. Nothing can tend more to obscure the force of such reasoning than to represent each argument as a *separate* Syllogism; for each circumstance taken separately is often weak, and it is only when the whole are taken together that the evidence becomes strong. However, it often happens that there is a real defect in the chain. Sometimes the proof of some of the circumstances fails. Sometimes the whole chain taken together might very well concur in more than one individual: or finally, the accused may be able to refer to another chain of circumstances tending to prove either that he was not the murderer or that some one else was. These are just the defects to which Inductions also are liable. The stress of an Induction always lies in this, that if the connexion between B and C was merely casual and not the result of a general law,*

* But may there not be such a thing as a law which is general and yet not absolutely universal? This is admittedly true of derivative laws. May it not also be true of some laws which are not derivative?

there could not be so many and varied instances in which a B proved to be a C as are contained in our enumeration. But occasionally it will be found on inquiry that some of the alleged instances have not been proved—that all of them, even in conjunction, might have been the result of chance, or that there are other instances leading us to believe that there is no such general law—that, for example, some Bs (which are not positively known either to be Cs or not-Cs) are Xs, and that no Xs are known to be Cs. There is, moreover, another chance of error here, namely that we may have mistaken the law in question. It may appear on inquiry that all the Bs we have examined were Ds (though in point of fact some Bs are not Ds), and that the true law is not that All Bs are Cs, but that All Bs which are Ds are Cs. Questions of this kind, however, do not fall within the province of Logic. Indeed much of the present Treatise is liable to the same objection; but I hope the context has sufficiently shown where I have dealt with a topic, either in order to show that it does *not* belong to the province of Logic, or to give the reader correct ideas on some of the subjects usually (though in my opinion improperly) introduced into their Treatises by Logical writers, and which many of my readers have therefore met with.

THE END.

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